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A Life Cycle Optimization Approach to Hydrocarbon Recovery

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A Life Cycle Optimization Approach to Hydrocarbon Recovery

by

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Thesis

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Dedication

To my grandmother Emilia

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2010

Abstract

A Life Cycle Optimization Approach to Hydrocarbon Recovery

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The objective of reservoir management is to maximize a key performance indicator (net present value in this study) at a minimum cost. A typical approach includes engineering analysis, followed by the economic value of the technical study. In general, operators are inclined to spend more effort on the engineering side to the detriment of the economic area, leading to unbalanced and occasionally suboptimal results. Moreover, most of the optimization methods used for production scheduling focus on a given recovery phase, or medium-term strategy, as opposed to an integrated solution that allocates resources from discovery to field abandonment.

This thesis addresses the optimization of a reservoir under both technical and economic constraints. In particular, the method presented introduces a life cycle maximization approach to establish the best exploitation strategy throughout the life of the project. Deterministic studies are combined with stochastic modeling and risk analysis to assess decision making under uncertainty. To demonstrate the validity of the model, this document offers two case studies and the optimal times associated with each recovery phase.

In contrast with traditional depletion strategies, where the optimization is done myopically by maximizing the NPV at each recovery phase, our results suggest that time is dramatically reduced when the net present value is optimized globally by maximizing the NPV for the life of the project. Furthermore, the sensitivity analysis proves that the original oil in place and non-engineering parameters such as the price of oil are the most influential variables. The case studies clearly show the greater economic efficiency of this life cycle approach, confirming the potential of this optimization technique for practical reservoir management.

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Chapter 1: Introduction

1.1 BACKGROUND

Mathematically, optimization involves finding the extreme values of a function. Given a function of several variables, an optimization scheme will find the combination of these variables that produces an extreme value in the function. In our case, the net present value (NPV) provides an operator's expected returns on the basis of different variables. Numerical optimization of the function will determine the mix of parameters that yields the maximum expected return.

Optimization methods are the cornerstone of operations research, a field originated before World War II with the aim of improving decision making and efficiency. According to Carroll and Horne**,** operations research concepts were not adopted by the petroleum industry until the early 1950's. Since then, the majority of the optimizing methods described in the literature involved linear programming techniques applied to reservoir and production management. Although previous research shares capital investment efficiency as the key indicator to allocate resources, the focus is to model field operations in the short, medium term, and occasionally over the long term. However, none of these models aims to optimize the NPV over the entire life of the project based on recovery efficiency processes and their associated stream of cash.

Historically, industry practices have approached the issue of implementing new recovery mechanisms when the returns from previous phases, for instance primary or secondary recovery, start to decrease and the revenues barely offset the costs. Therefore, there is hardly any evidence for successful applications of life cycle optimization methods for a reservoir over its productive life.

1.2 MOTIVATION FOR THE RESEARCH

The motivation for this thesis began with what is, in theory, a straight forward question: when is the best time to start enhanced oil recovery (EOR)? Virtually all the research in this area focuses on modeling reservoir performance in an attempt to minimize the financial risks associated with EOR. A different approach, possibly the opposite, is to use economic optimization methods by simplifying the reservoir behavior. The aim of this thesis is to develop a decision-making framework on reservoir exploitation over the life of the project, based purely on global economic optimization. The advantage of this approach is that it introduces decision questions of a broader nature to obtain global solutions. As the base case and the case study will show, it is reasonable to assume that the simplification of the reservoir behavior can be tolerated and still produce quality decisions.

1.3 OBJECTIVES OF THE THESIS

The objective of this study is to develop a systematic optimization method to determine the time that must be devoted to each recovery phase to maximize the NPV of the project over its life. Meeting this objective involves two steps: establish and optimization model and evaluate the model.

1.3.1 Step 1: Establish an Optimization Model

Optimization requires defining an objective function and the corresponding decision variables. The objective function used in this research was the NPV of a time series of cash flows generated per recovery phase over the life of the project. The decision variables were the optimal time that should be assigned to each recovery phase.

1.3.2 Step 2: Evaluate the Optimization Model

Once the optimization model was defined, it was tested against actual field data though a base case. Next, the robustness of the model was evaluated using two methods: deterministic and stochastic.

1.4 DOCUMENT LAYOUT

The thesis is organized into 3 parts. Part A presents a description of the mathematical formulation of the model. A numerical approach to this formulation is introduced in chapter 2, while the analytical approach is covered in chapter 3. Both approaches are evaluated using a generic example.

In part B the method is tested with a deterministic evaluation of the model and in part C with a stochastic evaluation. Part B includes chapters 4. Chapter 4 applies a deterministic approach to a base case.

Part C shows a stochastic approach to the optimization problem within a decision analysis framework in chapter 5 and with a Monte Carlo simulation in chapter 6. Chapter 7 reviews the conclusions and future work. Finally, appendix A discusses a case study in west Texas.

With the exception of appendix A, the base case and variations of this data are used to test the model. Additionally, the optimizations from chapters 1 to 7 are done myopically, by determining the maximum NPV at each recovery phase, and globally, by obtaining the maximum NPV of all the phases simultaneously.

PART A: *FORMULAE*

Chapter 2: Numerical Approach

2.1 INTRODUCTION

Optimization requires defining an objective function and the corresponding decision variables. The objective function used in this research was the net present value (NPV) of a series of cash flows. The decision variables were the time that should be assigned to each recovery phase to maximize the NPV. The NPV of the cash flows was obtained by discounting the net income stream (total revenue minus total expenses) over the life of the project back to the present.

To achieve optimization on the reservoir performance, the recovery efficiency history was introduced into the objective function. Reservoir performance was reproduced assuming an exponential decline model.

The purpose of this chapter is to introduce the numerical formulation of the model and its components.

2.2 MODELING RECOVERY EFFICIENCY

The first step of this research was to develop a model for reservoir performance. The goal was to provide a simple but functional reservoir model to include in a larger equation. We adopted an exponential decline model to replicate the reservoir dynamics.

2.2.1 Exponential Decline

The exponential decline model provides a framework to determine recovery efficiency. Recovery efficiency is defined as the recovered amount of hydrocarbons, expressed as a percentage of total hydrocarbons originally in place. Recovery efficiency is a function of the process occurring and depends on permeability, production mechanisms, etc. Lake (2010) takes the recovery efficiency for an exponential decline to be:

$$E_R(t) = E_R^0 + (E_R^\infty - E_R^0)\left(1 - e^{-t/\tau}\right)$$

Equation 1

where

E_R^0	=	recovery efficiency at time zero
E_R^∞	=	theoretical ultimate recovery efficiency
τ	=	time constant for production

The theoretical ultimate recovery efficiency and the time constant for production depend strongly on the reservoir properties and how the reservoir is produced. Although both parameters can change with time, we assume they are constant for each recovery phase.

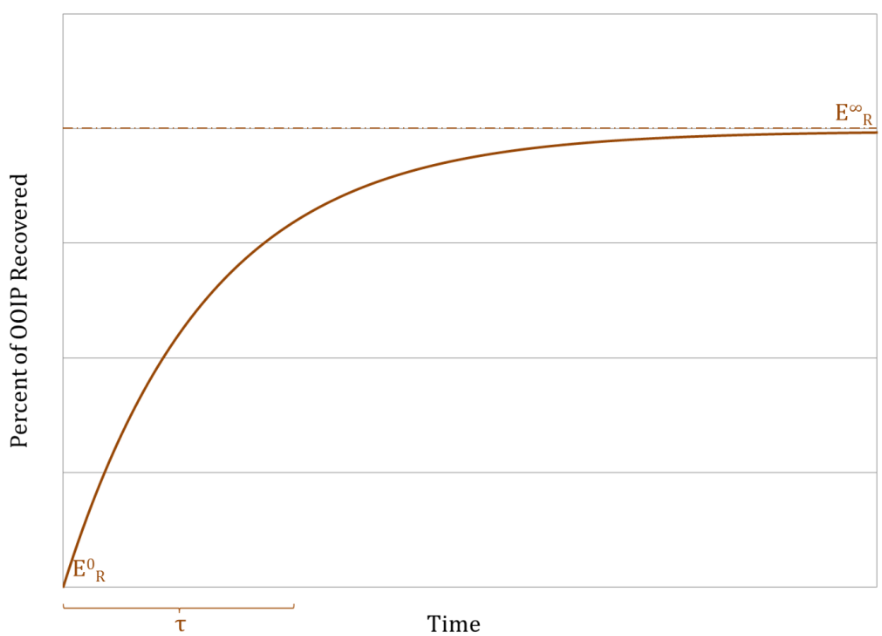

Figure 1: Typical recovery efficiency history for exponential decline

2.2.2 Definitions

By recovery phase we refer to the different production stages a petroleum reservoir undergoes over its life. The recovery phases discussed in this text include primary, secondary and tertiary recovery.

2.2.2.1 Primary Recovery

Primary recovery is the production stage in which natural reservoir energy displaces hydrocarbons from the reservoir into the wellbore. According to Lake and Walsh (2003) *"the original reservoir energy brings the fluids to the wellbore even though external energy (artificial lift) may be needed to bring the fluids to the surface."*

Although primary recovery is the most economical form of production only a small percentage of the original oil in place (OOIP), around 12 to 15% (Lake and Walsh), is produced during this phase.

2.2.2.2 Secondary Recovery

Secondary recovery is the production stage in which an external fluid is injected into the reservoir through injector wells. The most common fluids are water and natural gas. Lake and Walsh specified that *"other common but less likely injected fluids include enriched hydrocarbon gases, nitrogen, flue gas, carbon dioxide, steam, water-soluble surfactants, and water-soluble polymers."* Approximately 15 to 20% of the OOIP is produced through secondary recovery (Lake and Walsh).

2.2.2.3 Tertiary Recovery

Tertiary recovery, also known as enhanced oil recovery (EOR), starts when a second fluid is injected after secondary production. *"Most commercial oil reservoirs undergo primary and secondary recovery, but only a few as yet undergone tertiary* *recovery owning to economic limitations. Fluids injected for tertiary recovery include carbon dioxide, enriched hydrocarbon gases, and polymer and surfactant solutions"* (Lake and Walsh). The average tertiary recovery as a percentage of OOIP is 4 to 11% (NIPER, 1986).

2.2.3 Numerical Phase Modeling

2.2.3.1 Primary Recovery

Equation 1 for primary production becomes

$$E_{R1}(t) = E_{R1}^{0} + (E_{R1}^{\infty} - E_{R1}^{0})\left(1 - e^{-t/\tau_1}\right)$$

Equation 2

where

E_{R1}^{0}	=	primary recovery efficiency at time zero
E_{R1}^{∞}	=	theoretical ultimate recovery efficiency for primary recovery
τ_1	=	primary recovery time constant for production

Note that E_{R1}^{0} for primary recovery is zero; therefore, Equation 2 can be simplified

$$E_{R1}(t) = E_{R1}^{\infty}\left(1 - e^{-t/\tau_1}\right)$$

Equation 3

2.2.3.2 Secondary Recovery

Equation 1 for secondary production becomes

$$E_{R2}(t) = E_{R1}\left(t_{Life1}\right) + \left[E_{R2}^{\infty\prime} - E_{R1}\left(t_{Life1}\right)\right]\left(1 - e^{-\left(t - t_{Life1}\right)/\tau_2}\right) for\ t \geq t_{Life1}$$

Equation 4

where

$E_{R1}\left(t_{Life1}\right)$	=	recovery efficiency at the end of primary production
t_{Life1}	=	total years of primary production
$E_{R2}^{\infty\prime}$	=	theoretical ultimate recovery efficiency for secondary recovery

τ_2 = secondary recovery time constant for production

We take the theoretical ultimate recovery efficiency for secondary recovery to be

$$E_{R2}^{\infty\prime} = E_{R1}(t_{Life1}) + \Delta E_{R2}^{\infty}$$

$$E_{R2}^{\infty\prime} = E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right) + (E_{R2}^{\infty} - E_{R1}^{\infty})$$

Equation 5

where t_{life1} is one of the decision variables used to optimize the NPV.

2.2.3.3 Tertiary Recovery

Similarly Equation 1 for tertiary production becomes

$$E_{R3}(t) = E_{R2}(t_{Life2}) + \left[E_{R3}^{\infty\prime} - E_{R2}(t_{Life2})\right]\left(1 - e^{-(t - t_{Life1} - t_{Life2})/\tau_3}\right) \; for \; t \geq t_{Life2}$$

Equation 6

where

$E_{R2}(t_{Life2})$ = recovery efficiency at the end of secondary production

t_{Life2} = total years of secondary production

$E_{R3}^{\infty\prime}$ = theoretical ultimate recovery efficiency for tertiary recovery

τ_3 = tertiary recovery time constant for production

We take the theoretical ultimate recovery efficiency for tertiary recovery to be

$$E_{R3}^{\infty\prime} = E_{R2}(t_{Life2}) + \Delta E_{R3}^{\infty}$$

or

$$\boldsymbol{E_{R3}^{\infty\prime} = E_{R1}(t_{Life1}) + \left[E_{R2}^{\infty\prime} - E_{R1}(t_{Life1})\right]\left(1 - e^{-t_{Life2}/\tau_2}\right) + (E_{R3}^{\infty} - E_{R2}^{\infty})}$$

Equation 7

where t_{life2} is one of the decision variables used to optimize the NPV. Note that $t_{Life2} = t_{Life1} - t_1$.

2.2.3.4 n Recovery Phases

Equation 1 for n recovery phase becomes

$$\boldsymbol{E_{R(n)}(t) = E_{R(n-1)}\left(t_{Life(n-1)}\right) + \left[E_{R(n)}^{\infty\prime} - E_{R(n-1)}\left(t_{Life(n-1)}\right)\right]\left(1 - e^{-\left(t - \sum_1^{n-1} t_t\right)/\tau_n}\right)}$$

$$\textbf{for } \boldsymbol{t \geq t_{Life(n-1)}}$$

Equation 8

where

$E_{R(n-1)}\left(t_{Life(n-1)}\right)$ = recovery efficiency at the end of n-1 production

$E_{Rn}^{\infty\prime}$ = theoretical ultimate recovery efficiency for n recovery

τ_n = n recovery time constant for production

$t_{Life(n-1)}$ = total years of n-1 production

We take the theoretical ultimate recovery efficiency for n recovery phases to be

$$E_{Rn}^{\infty\prime} = E_{R(n-1)}\left(t_{Life(n-1)}\right) + \Delta E_{Rn}^{\infty}$$

or

$$E_{Rn}^{\infty\prime} = E_{R(n-2)}\left(t_{Life(n-2)}\right) + \left[E_{R(n-1)}^{\infty\prime} - E_{R(n-2)}\left(t_{Life(n-2)}\right)\right]\left(1 - e^{-\left(t_{Life(n-1)}\right)/\tau_{(n-1)}}\right) + \left(E_{Rn}^{\infty} - E_{R(n-1)}^{\infty}\right)$$

Equation 9

where $\mathrm{t_{Life(n-1)}}$ is one of the decision variables used to optimize the NPV. Note that $t_{Life(n-1)} = t - t_{Life(n-2)}$.

2.3 SELECTING AN ECONOMIC INDICATOR

The key to reservoir management's ability to optimize economic performance is the appropriate choice of a performance metric or economic indicator. According to Barua et al. (1986) the economic criterion is used as a measure of success. Numerical optimization of this performance metric will determine the mix of parameters that yields the maximum returns.

There are different economic indicators, or output measures, derived from the cash flow stream. Primarily these are the net present value (NPV), the project's internal rate of return (IRR), and the present value index, or profit investment ratio. The economic criterion used in this research is the NPV.

2.3.1 Internal Rate of Return (IRR)

The internal rate of return is the percentage rate that will discount a net cash flow stream to a cumulative present value of zero. The IRR can be thought of as the interest rate that money invested in a project will earn over the life of the project, or as the maximum rate of interest for project financing that will allow repayment with interest from net cash flow.

$$NPV = \sum_{0}^{t} \frac{CF_t}{(1 + IRR)^t} = 0$$

Equation 10

The IRR is a widely used indicator; however, selecting the IRR as an optimization criterion for reservoir projects can be misleading. Very high rates of return may overemphasize the importance of cash flow in the early years. As Barua et al. proves for different optimization runs, if a project can be designed to return profits rapidly it might be ranked higher than a longer project with smaller IRR even though the NPV of the first could be several times smaller than the last. Additionally, IRR does not provide any information about the materiality of a project. For these reasons the NPV is the better choice.

2.3.2 Net Present Value (NPV)

2.3.2.1 Definition

The net present value is the sum of a net cash flow stream when discounted at a specified rate. Typically, this rate equates the real[1] company's cost of capital, and thus NPV measures the value added by a project in excess of the real cost of capital or discount rate in real terms. Hence,

$$NPV_i = \sum_{t_{Life(i-1)}}^{t_{Lifei}} PV_t = \sum_{t_{Life(i-1)}}^{t_{Lifei}} \frac{CF_{it}}{(1+R)^t}$$

Equation 11

where

NPV_i	=	net present value of recovery phase i
PV_t	=	present value at time t
t_{Lifei}	=	time length of recovery phase i
CF_{it}	=	net cash flow in real terms for recovery phase i at time t
R	=	discount rate in real terms

The NPV will decrease as the discount rate increases. At some discount rate, the NPV will equal zero, this is the IRR or $NPV|_{R=IRR}=0$.

2.3.2.2 Discount Rate

The discounting scheme used in the model assumes that the company's cost of capital and the expected inflation rate are both constant over the life of the project for all the production phases. Specifying the cost of capital and the inflation

[1] "Real" indicates that the effects of inflation have been removed.

rate, we can express the discount rate in real terms by coupling the effects of both into one discount factor.

$$(1 + R) = \frac{(1 + C_C)}{(1 + i)}$$

Equation 12

where

C_C = cost of capital

i = inflation rate

Therefore, we refer to R as the discount rate in real terms since the effects of the inflation have been accounted for. Even without inflation (*i=0*), money would still have a time value. Money held now can be invested and interest earned, so that it becomes more valuable than money received at some future date. The time value of money is taken into account by discounting the cash flow.

A company's cost of capital is usually calculated as a weighted average of the cost of equity and the cost of debt, also known as the weighted average cost of capital (WACC). The WACC is used by companies to set the discount rates.

For the purposes of this research we are going to take the cost of capital to be 10%. The inflation rate will be set at 2.8%, the average value from 1914 to 2010. Substituting these percentages in Equation 12 results in

$$(1 + R) = \frac{(1 + 0.1)}{(1 + 0.028)}$$

$$R = 1.07 - 1 = 0.07$$

Equation 13

Unless otherwise stated, cash flow is discounted at 7% throughout this study. Note that we use a discount rate in real terms because all the costs in this research are model in real terms.

2.3.2.3 Cash Flow Components

The NPV is the sum of the present values of individual cash flows. The net cash flow in real terms at time t (CF_{it}) is equivalent to the inflow minus the outflow at the same time period t.

$$CF_{it} = Inflow_{it} - Outflow_{it}$$

Equation 14

The inflow is the result of

$$Inflow_{it} = N\$_{Oil}[E_{Ri}(t) - E_{Ri}(t-1)]$$

Equation 15

where

N	=	original oil in place in standard volume
$\$_{Oil}$	=	unit oil price (assume constant)[2]
$E_{Ri}(t)$	=	recovery efficiency of phase i at time t

and the outflow

$$Outflow_{it} = \$_{Capex(i)} + \$_{Opex(i)}$$

Equation 16

where

$\$_{Capex(i)}$	=	capital costs of recovery phase i
$\$_{Opex(i)}$	=	operating costs of recovery phase i

Capital costs should include those from drilling, facilities, pipelines, work over and any other capital expenditure incurred as a result of developing the field through recovery phase i. Costs and prices were used with no escalation. Any

[2] Refer to Liying Xu Ph.D. dissertation for "Application of Real Options to the Valuation and Decision Making in the Petroleum E&P Industry"

adjustments due to increases over time were assumed to be covered by the inflation rate.

2.3.3 NPV Phase Modeling

2.3.3.1 NPV for Primary Recovery

Equation 11 for primary recovery becomes

$$NPV_1 = \sum_{t=0}^{t_{Life1}} \frac{CF_{1t}}{(1+R)^t}$$

Equation 17

where the net cash flow (Equation 14) for primary recovery equals

$$CF_{1t} = N\$_{Oil}[E_{R1}(t) - E_{R1}(t-1)] - \left(\$_{Capex1} + \$_{Opex1}\right)$$

Equation 18

and

CF_{1t}	=	net cash flow in real terms for primary recovery at time t
$\$_{Capex1}$	=	capital costs of primary recovery
$\$_{Opex1}$	=	operating costs of primary recovery

2.3.3.2 NPV for Secondary Recovery

Equation 11 for secondary recovery becomes

$$NPV_2 = \sum_{t_{Life1}}^{t_{Life2}} \frac{CF_{2t}}{(1+R)^t}$$

Equation 19

where the net cash flow (Equation 14) for secondary recovery equals

$$CF_{2t} = N\$_{Oil}[E_{R2}(t) - E_{R2}(t-1)] - \left(\$_{Capex2} + \$_{Opex2}\right)$$

Equation 20

and

CF_{2t} = net cash flow in real terms for secondary recovery at time t

$\$_{Capex2}$ = capital costs of secondary recovery

$\$_{Opex2}$ = operating costs of secondary recovery

2.3.3.3NPV for Tertiary recovery

Equation 11 for tertiary recovery becomes

$$NPV_3 = \sum_{t_{Life2}}^{t_{Life3}} \frac{CF_{3t}}{(1+R)^t}$$

Equation 21

where the net cash flow (Equation 14) for tertiary recovery equals

$$CF_{3t} = N\$_{Oil}[E_{R3}(t) - E_{R3}(t-1)] - \left(\$_{Capex3} + \$_{Opex3}\right)$$

Equation 22

and

CF_{3t} = net cash flow in real terms for tertiary recovery at time t

$\$_{Capex3}$ = capital costs of tertiary recovery

$\$_{Opex3}$ = operating costs of tertiary recovery

2.3.3.4NPV for n^th Recovery Phases

Equation 11 for n recovery phase becomes

$$NPV_n = \sum_{t_{Life(n-1)}}^{t_{Lifen}} \frac{CF_{nt}}{(1+R)^t}$$

Equation 23

where the net cash flow (Equation 14) for n recovery phase equals

$$CF_{nt} = N\$_{Oil}[E_{Rn}(t) - E_{Rn}(t-1)] - \left(\$_{Capexn} + \$_{Opexn}\right)$$

Equation 24

and

CF_{nt} = net cash flow in real terms for n recovery at time t

$\$_{Capexn}$= capital costs of n recovery phase

$\$_{Opexn}$ = operating costs of n recovery phase

2.4 ECONOMIC OPTIMIZATION METHOD

2.4.1 Objective Function and Decision Variables

Optimization involves finding the values for the decision variables that maximize the objective function. As shown in previous sections, the objective function to be maximized is the NPV, and the decision variables are the optimal times that should be assigned to each recovery phase.

Although the formulas derived in part one can be applied to any unit of time, we have used years throughout the text. Therefore, all the time related parameters, mainly τ (time constant for production), R (discount rate in real terms) and the decision variables t_{Life1}, t_{Life2}, and t_{Life3}, are given in years.

This research approaches optimization in two different ways: myopic and life cycle optimization.

2.4.2 Myopic Optimization

Traditionally, industry practices have attempted to maximize the NPV with short to medium-term strategies. Usually, optimization takes place over a limited time period within a recovery phase. A common optimization strategy focuses on a 2-5 year window. To mimic this strategy and to compare the results to a long-term approach every analysis includes a myopic optimization. The aim of the myopic optimization is to maximize the NPV per recovery phase or until the economic limit is reached.

This is equivalent to finding the value of t that makes the derivative of the NPV a function of time equal to zero. Production beyond this point would lead to losses because revenues would no longer offset costs (refer to **Figure 2**).

$$\frac{dNPV_i}{dt} = 0$$

Equation 25

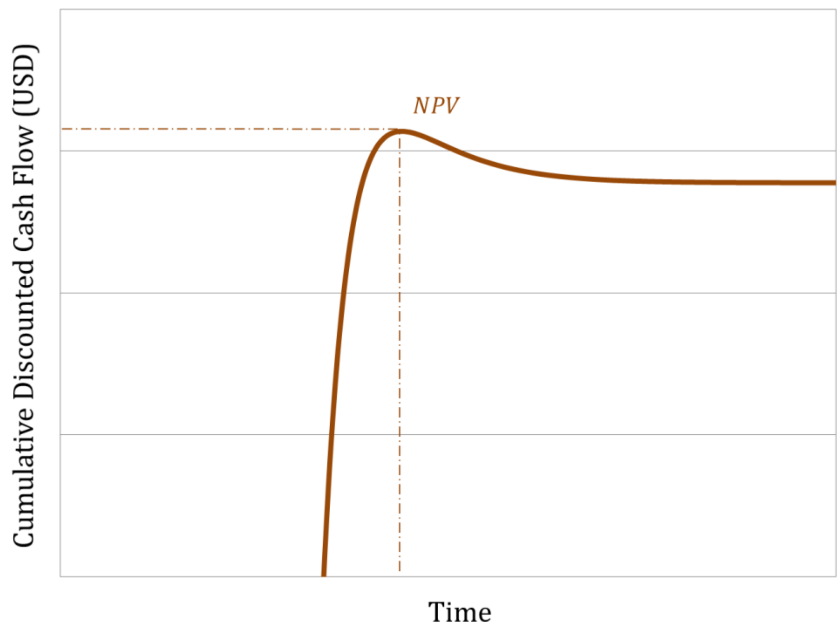

Figure 2: Myopic optimization.

2.4.3 Life Cycle Optimization

A different approach to a myopic optimization would be a long-term strategy or a life cycle optimization. The objective of a life cycle optimization is to determine the time that must be devoted to each recovery phase to maximize the NPV of the project over its life. We achieve this objective by finding the values for the decision variables (t_{Life1}, t_{Life2}, and t_{Life3}) that maximize the total NPV.

$$Max\left(\sum_{i=1}^{n} NPV_i\right) = Max(NPV_1 + NPV_2 + NPV_3)$$

Equation 26

As **Figure 3** shows, only production beyond the optimal time during the last recovery phase would lead to a smaller NPV using a life cycle optimization strategy. Note that t_{switch}, or the time in which a new production phase starts, is substantially before the economic limit.

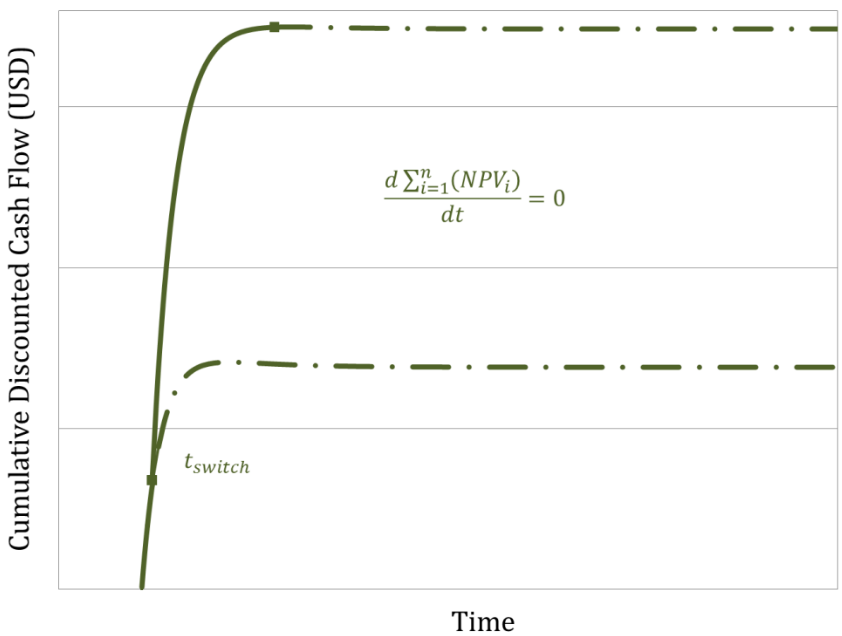

Figure 3: Life cycle optimization

2.5 ILLUSTRATIVE EXAMPLE

2.5.1 Problem Statement and Assumptions

Figure 4 shows the recovery efficiency as a function of time for a field in south central Wyoming. Assume the oil price is $55 per barrel and the operational costs per recovery phase are assessed according to **Table 1**.

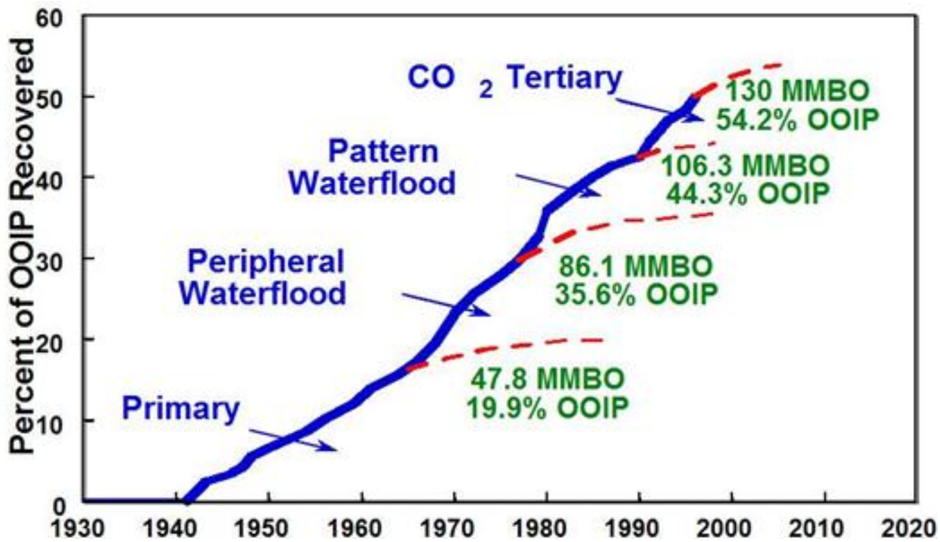

Figure 4: Recovery efficiency as a function of time. From Brokmeyer et al., 1996

Table 1: Cost estimates

	Primary	Peripheral Waterflood	Pattern Waterflood	Tertiary
Opex ($/bbl)	3	7	8	12

Using the numerical approach derived in this chapter:

a. Maximize the NPV per recovery phase. Compute the total NPV for this myopic approach. What would be the values for t_{Life1}, t_{Life2} and t_{Life3}?

b. Determine the total NPV according to **Figure 4** production profile (base case).

c. Plot the recovery efficiency and the NPV as a function of time for parts a and b.

2.5.2 Data Fit

The first step is to find the best-fit values for the recovery time constants for production τ_1, τ_2, and τ_3. Note that τ_1, τ_2, and τ_3, are treated as adjustable parameters. Therefore, the recovery efficiency formula for primary production, Equation 3, is used to calculate percent of original oil in place (OOIP) recovered after 1 year.

$$E_{R1}(t=1) = E_{R1}^{\infty}\left(1 - e^{-t/\tau_1}\right) = 0.199\left(1 - e^{-1/\tau_1}\right)$$

From **Figure 4** we can estimate primary production to last approximately 24 years, from 1942 to 1965. Plotting actual and calculated data for that interval and using a trial-and-error procedure, τ_1 is set at around 10 years.

Secondary production follows for 13 years (1966-1978). From Equation 4 peripheral waterflooding at any time $t \geq t_{Life1}$ is equal to

$$E_{R2}(t) = E_{R1}\left(t_{Life1}\right) + \left[E_{R2}^{\infty\prime} - E_{R1}\left(t_{Life1}\right)\right]\left(1 - e^{-\left(t - t_{Life1}\right)/\tau_2}\right) \; for\ t \geq t_{Life1}$$

where

$$\boldsymbol{E_{R1}\left(t_{Life1} = 24\right) = 0.199\left(1 - e^{-24/10}\right) = 18.09\%}$$

$$\boldsymbol{E_{R2}^{\infty\prime} = E_{R1}\left(t_{Life1}\right) + \left(E_{R2}^{\infty} - E_{R1}^{\infty}\right) = 0.181 + (0.356 - 0.199) = 33.79\%}$$

Plotting actual versus calculated data and using a trial-and-error procedure, $\tau_{2Peripheral}$ is set at around 5 years. Pattern waterflooding starts in 1979 and ends 10 years later. At the beginning of 1990 CO_2 replaces waterflooding for 8 years until 1998. Following the same procedure as for primary and peripheral we take $\tau_{2Pattern}$ and $\tau_{3Tertiary}$ to be 5 and 7 years, respectively.

Figure 5 shows a comparison between the actual and the calculated data. We can easily calculate original oil in place (OOIP) with the recovery efficiency and the volumes provided in **Figure 4** as

$$47.8\ MMBO/0.199 = 240.20\ MMBO$$

$$86.1\ MMBO/0.356 = 241.85\ MMBO$$

$$106.3\ MMBO/0.443 = 239.95\ MMBO$$

$$130\ MMBO/0.542 = 239.85\ MMBO$$

or

$$OOIP = 240\ MMBO$$

Table 2 displays the recovery efficiency calculations per year according to the formulae provided in previous sections.

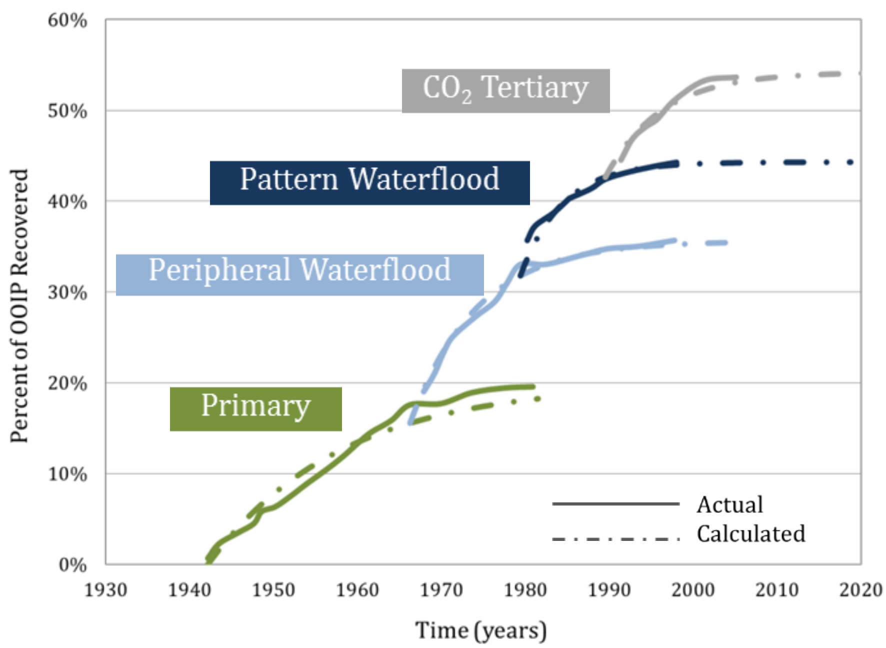

Figure 5: Actual and calculated recovery efficiency as a function of time

Table 2: Summary of calculated recovery efficiencies as a function of time

t_1 (years)	E_{R1} (t) (fraction)	t_{2Per} (years)	E_{R2Per} (t) (fraction)	t_{2Pat} (years)	E_{R2Pat} (t) (fraction)	t_3 (years)	E_{R3} (t) (fraction)
0	0.000	24	0.181	37	0.326	47	0.402
1	0.019	25	0.209	38	0.342	48	0.415
2	0.036	26	0.233	39	0.355	49	0.426
3	0.052	27	0.252	40	0.366	50	0.436
4	0.066	28	0.267	41	0.374	51	0.445
5	0.078	29	0.280	42	0.381	52	0.452
6	0.090	30	0.291	43	0.387	53	0.458
7	0.100	31	0.299	44	0.392	54	0.464
8	0.110	32	0.306	45	0.396	**55**	**0.469**
9	0.118	33	0.312	46	0.399		
10	0.126	34	0.317	**47**	**0.402**		
11	0.133	35	0.321				
12	0.139	36	0.324				
13	0.145	**37**	**0.326**				
14	0.150						
15	0.155						
16	0.159						
17	0.163						
18	0.166						
19	0.169						
20	0.172						
21	0.175						
22	0.177						
23	0.179						
24	**0.181**						

2.5.3 Solution to Illustrative Example

a. Maximize the NPV per recovery phase. Compute the total NPV for this myopic approach. What would be the values for t_{Life1}, t_{Life2} and t_{Life3}?

2.5.3.1 Estimating Opex per Year

To determine the NPV per recovery phase, we must estimate first the operating cost per year.

The simplest approach would be to subtract from the oil price the operating cost per barrel. This approach is equivalent to the assumption that all the operating costs are variable or directly proportional to the number of barrels produced. However, this approach can be misleading. Most operating costs related with production operations are fixed or independent on the number of barrels produced. Maintenance routines and crews, inventories, etc. are budgeted based on expected production during the first years of operations. To reflect this practice in our calculations we set the operating costs per year equal to the average production of the first 10 years times the operating cost per barrel.

Production for year t is

$$Production(t) = N[E_{R1}(t) - E_{R1}(t-1)]$$

Equation 27

and substituting for year 1 we have

$$Production(1) = 2.4 \cdot 10^8 bbls[0.0189 - 0] = 4.5M\ barrels$$

Table 3 shows production for years 1 through 10 and **Table 4** the yearly operating cost per recovery phase.

Table 3: Average production

t (years)	E_{R1} (t) (fraction)	ΔE_{R1} (fraction)	$N\Delta E_R$ (bbls)
0	0.000	-	-
1	0.019	0.019	4,544,965
2	0.036	0.017	4,112,454
3	0.052	0.016	3,721,103
4	0.066	0.014	3,366,993
5	0.078	0.013	3,046,581
6	0.090	0.011	2,756,661
7	0.100	0.010	2,494,330
8	0.110	0.009	2,256,963
9	0.118	0.009	2,042,184
10	0.126	0.008	1,847,845
Note: N = 240,000		Average	3,019,008

Table 4: Yearly operating cost per recovery phase

Recovery Phase$_i$	Opex ($/bbl)	$\$_{Opexi}$
1. Primary	3	$9,057,023
2. Secondary Peripheral	7	$21,133,055
3. Secondary Pattern	8	$24,152,062
4. Tertiary	12	$36,228,093

2.5.3.2 Estimating NPV

The contribution to the NPV from primary recovery is determined using Equation 17 and Equation 18 where the cash flow for year 1 is

$$CF_1(1) = (2.4 \cdot 10^8 bbls)(\$55/bbl)[0.0189 - 0] - 9{,}057{,}023 = \$240M$$

$$PF_1(1) = \frac{\$240M}{(1 + 0.07)^1} = \$225M$$

Table 5 shows the NPV for primary recovery as a function of time.

Table 5: Summary of calculated NPV for primary recovery as a function of time (myopic optimization)

PRIMARY				
t (years)	$E_{R1}(t)$ (fraction)	$\Delta E_{R1}(t)$ (fraction)	$PV_1(t)$	NPV_1
0	0.000	0.000	$0	$0
1	0.019	0.019	$225,155,184	$225,155,184
2	0.036	0.017	$189,647,973	$414,803,157
3	0.052	0.016	$159,670,817	$574,473,974
4	0.066	0.014	$134,366,889	$708,840,863
5	0.078	0.013	$113,011,829	$821,852,692
6	0.090	0.011	$94,993,286	$916,845,978
7	0.100	0.010	$79,793,613	$996,639,592
8	0.110	0.009	$66,975,239	$1,063,614,830
9	0.118	0.009	$56,168,294	$1,119,783,124
10	0.126	0.008	$47,060,152	$1,166,843,277
11	0.133	0.007	$39,386,585	$1,206,229,862
12	0.139	0.006	$32,924,279	$1,239,154,140
13	0.145	0.006	$27,484,514	$1,266,638,654
14	0.150	0.005	$22,907,816	$1,289,546,470
15	0.155	0.005	$19,059,433	$1,308,605,903
16	0.159	0.004	$15,825,514	$1,324,431,417
17	0.163	0.004	$13,109,874	$1,337,541,291
18	0.166	0.003	$10,831,264	$1,348,372,555
19	0.169	0.003	$8,921,057	$1,357,293,611
20	0.172	0.003	$7,321,296	$1,364,614,907
21	0.175	0.003	$5,983,041	$1,370,597,947
22	0.177	0.002	$4,864,973	$1,375,462,921
23	0.179	0.002	$3,932,215	$1,379,395,136
24	0.181	0.002	$3,155,329	$1,382,550,465
25	0.183	0.002	$2,509,478	$1,385,059,943
26	0.184	0.002	$1,973,707	$1,387,033,650
27	0.186	0.001	$1,530,347	$1,388,563,997
28	0.187	0.001	$1,164,496	$1,389,728,493
29	0.188	0.001	$863,598	$1,390,592,090
30	0.189	0.001	$617,071	$1,391,209,161
31	0.190	0.001	$416,004	$1,391,625,165
32	0.191	0.001	$252,897	$1,391,878,062
33	0.192	0.001	$121,436	$1,391,999,498
34	**0.192**	**0.001**	**$16,313**	**$1,392,015,811**

To maximize the NPV per recovery phase we select the last year in which the cash flow or present value is positive or equal to zero. Therefore, the myopic maximization of the NPV for primary recovery results in a production life of 34 years $\left(t_{Life1} = 34\ years\right)$.

Likewise, the contribution to the NPV from peripheral waterflooding is determined using Equation 19 and Equation 20 where the cash flow for year 35 is

$$\mathrm{CF_2(35)} = (2.4 \cdot 10^8 \mathrm{bbls})(\$55/\mathrm{bbl})[0.2208 - 0.1924] - 21{,}133{,}055 = \$355\mathrm{M}$$

$$\mathrm{PF_2(35)} = \frac{\$355\mathrm{M}}{(1+0.07)^{35}} = \$33M$$

Tables 6-8 show the calculated NPV for peripheral waterflooding, pattern waterflooding and CO_2, respectively.

Table 6: Summary of calculated NPV for peripheral waterflooding as a function of time (myopic optimization)

SECONDARY PERIPHERICAL

t (years)	$E_{R2Per}(t)$ (fraction)	$\Delta E_{R2}(t)$ (fraction)	PV_2	$NPV_{2Peripheral}$
34	0.192	0.001	$16,313	$1,392,015,811
35	0.221	0.028	$33,206,259	$1,425,222,070
36	0.244	0.023	$25,073,069	$1,450,295,140
37	0.263	0.019	$18,871,743	$1,469,166,882
38	0.279	0.016	$14,147,182	$1,483,314,064
39	0.292	0.013	$10,551,256	$1,493,865,321
40	0.302	0.010	$7,817,673	$1,501,682,994
41	0.311	0.009	$5,742,756	$1,507,425,750
42	0.318	0.007	$4,170,735	$1,511,596,484
43	0.323	0.006	$2,982,491	$1,514,578,975
44	0.328	0.005	$2,086,945	$1,516,665,921
45	0.332	0.004	$1,414,469	$1,518,080,390
46	0.335	0.003	$911,844	$1,518,992,234
47	0.338	0.003	$538,402	$1,519,530,636
48	0.340	0.002	$263,079	$1,519,793,715
49	**0.342**	**0.002**	**$62,150**	**$1,519,855,866**

Table 7: Summary of calculated NPV for pattern waterflooding as a function of time (myopic optimization)

SECONDARY PATTERN

t (years)	$E_{R2Pat}(t)$ (fraction)	$\Delta E_{R2}(t)$ (fraction)	PV_2	$NPV_{2Pattern}$
49	0.342	0.002	$62,150	$1,519,855,866
50	0.357	0.016	$6,246,983	$1,526,102,849
51	0.370	0.013	$4,641,096	$1,530,743,945
52	0.381	0.011	$3,421,409	$1,534,165,354
53	0.389	0.009	$2,496,634	$1,536,661,988
54	0.397	0.007	$1,796,962	$1,538,458,950
55	0.402	0.006	$1,269,013	$1,539,727,963
56	0.407	0.005	$871,974	$1,540,599,937
57	0.411	0.004	$574,652	$1,541,174,589
58	0.414	0.003	$353,205	$1,541,527,794
59	0.417	0.003	$189,420	$1,541,717,214
60	**0.419**	**0.002**	**$69,385**	**$1,541,786,600**

Table 8: Summary of calculated NPV for tertiary recovery as a function of time (myopic optimization)

TERTIARY

t (years)	$E_{R3}(t)$ (fraction)	$\Delta E_{R3}(t)$ (fraction)	PV_3	NPV_3
60	0.419	0.002	$69,385	$1,541,786,600
61	0.432	0.013	$2,221,450	$1,544,008,050
62	0.444	0.011	$1,727,050	$1,545,735,099
63	0.453	0.010	$1,331,259	$1,547,066,358
64	0.462	0.009	$1,015,047	$1,548,081,405
65	0.469	0.007	$763,016	$1,548,844,421
66	0.476	0.006	$562,712	$1,549,407,133
67	0.481	0.006	$404,060	$1,549,811,192
68	0.486	0.005	$278,916	$1,550,090,109
69	0.491	0.004	$180,698	$1,550,270,807
70	0.494	0.004	$104,087	$1,550,374,893
71	**0.497**	**0.003**	**$44,787**	**$1,550,419,680**

The total NPV for the myopic approach is equal to

$$\sum_{i=1}^{n} NPV_i = NPV_1 + NPV_{2Per} + NPV_{2Pat} + NPV_3 = \$1{,}550{,}243{,}893$$

and the production life for each recovery phase is

$$t_{Life1} = 34\ years$$

$$t_{Life2Per} = 15\ years$$

$$t_{Life2Pat} = 11\ years$$

$$t_{Life3} = 11\ years$$

b. Determine the total NPV according to **Figure 4** production profile (base case).

According to **Figure 4** primary production lasted 24 years, peripheral waterflooding 13, pattern waterflooding 10, and CO_2 8.

Hence, the total NPV for this development strategy is

$$\sum_{i=1}^{n} NPV_i = NPV_1 + NPV_{2Per} + NPV_{2Pat} + NPV_3 = \$1{,}702{,}637{,}541$$

The calculated NPV per year for the base case is presented in Tables 9-12.

Table 9: Summary of calculated NPV for primary recovery as a function of time (base case)

PRIMARY

t (years)	$E_{R1}(t)$ (fraction)	$\Delta E_{R1}(t)$ (fraction)	$PV_1(t)$	NPV_1
0	0.000	0.000	$0	$0
1	0.019	0.019	$225,155,184	$225,155,184
2	0.036	0.017	$189,647,973	$414,803,157
3	0.052	0.016	$159,670,817	$574,473,974
4	0.066	0.014	$134,366,889	$708,840,863
5	0.078	0.013	$113,011,829	$821,852,692
6	0.090	0.011	$94,993,286	$916,845,978
7	0.100	0.010	$79,793,613	$996,639,592
8	0.110	0.009	$66,975,239	$1,063,614,830
9	0.118	0.009	$56,168,294	$1,119,783,124
10	0.126	0.008	$47,060,152	$1,166,843,277
11	0.133	0.007	$39,386,585	$1,206,229,862
12	0.139	0.006	$32,924,279	$1,239,154,140
13	0.145	0.006	$27,484,514	$1,266,638,654
14	0.150	0.005	$22,907,816	$1,289,546,470
15	0.155	0.005	$19,059,433	$1,308,605,903
16	0.159	0.004	$15,825,514	$1,324,431,417
17	0.163	0.004	$13,109,874	$1,337,541,291
18	0.166	0.003	$10,831,264	$1,348,372,555
19	0.169	0.003	$8,921,057	$1,357,293,611
20	0.172	0.003	$7,321,296	$1,364,614,907
21	0.175	0.003	$5,983,041	$1,370,597,947
22	0.177	0.002	$4,864,973	$1,375,462,921
23	0.179	0.002	$3,932,215	$1,379,395,136
24	**0.181**	**0.002**	**$3,155,329**	**$1,382,550,465**

Table 10: Summary of calculated NPV for peripheral waterflooding as a function of time (base case)

SECONDARY PERIPHERICAL

t (years)	$E_{R2Per}(t)$ (fraction)	$\Delta E_{R2}(t)$ (fraction)	PV_2	$NPV_{2Peripheral}$
24	0.181	0.002	$3,155,329	$1,382,550,465
25	0.209	0.028	$65,321,738	$1,447,872,203
26	0.233	0.023	$49,322,522	$1,497,194,725
27	0.252	0.019	$37,123,574	$1,534,318,299
28	0.267	0.016	$27,829,649	$1,562,147,948
29	0.280	0.013	$20,755,918	$1,582,903,866
30	0.291	0.010	$15,378,546	$1,598,282,412
31	0.299	0.009	$11,296,870	$1,609,579,282
32	0.306	0.007	$8,204,467	$1,617,783,749
33	0.312	0.006	$5,867,011	$1,623,650,760
34	0.317	0.005	$4,105,337	$1,627,756,097
35	0.321	0.004	$2,782,475	$1,630,538,572
36	0.324	0.003	$1,793,735	$1,632,332,307
37	**0.326**	**0.003**	**$1,059,119**	**$1,633,391,426**

Table 11: Summary of calculated NPV for pattern waterflooding as a function of time (base case)

SECONDARY PATTERN

t (years)	$E_{R2Pat}(t)$ (fraction)	$\Delta E_{R2}(t)$ (fraction)	PV_2	$NPV_{2Pattern}$
37	0.326	0.003	$1,059,119	$1,633,391,426
38	0.342	0.016	$14,069,403	$1,647,460,829
39	0.355	0.013	$10,452,638	$1,657,913,467
40	0.366	0.011	$7,705,668	$1,665,619,135
41	0.374	0.009	$5,622,898	$1,671,242,033
42	0.381	0.007	$4,047,103	$1,675,289,136
43	0.387	0.006	$2,858,059	$1,678,147,195
44	0.392	0.005	$1,963,853	$1,680,111,049
45	0.396	0.004	$1,294,227	$1,681,405,275
46	0.399	0.003	$795,486	$1,682,200,761
47	**0.402**	**0.003**	**$426,610**	**$1,682,627,371**

Table 12: Summary of calculated NPV for tertiary recovery as a function of time (base case)

TERTIARY				
t (years)	$E_{R3}(t)$ (fraction)	$\Delta E_{R3}(t)$ (fraction)	PV_3	NPV_3
47	0.402	0.003	$426,610	$1,682,627,371
48	0.415	0.013	$5,353,351	$1,687,980,722
49	0.426	0.011	$4,161,922	$1,692,142,644
50	0.436	0.010	$3,208,127	$1,695,350,771
51	0.445	0.009	$2,446,105	$1,697,796,876
52	0.452	0.007	$1,838,751	$1,699,635,627
53	0.458	0.006	$1,356,048	$1,700,991,675
54	0.464	0.006	$973,722	$1,701,965,396
55	**0.469**	**0.005**	**$672,145**	**$1,702,637,541**

c. Plot the recovery efficiency and the NPV as a function of time for parts a and b.

The recovery efficiencies for parts a and b are plotted in **Figure 6** and the NPV as a function of time in **Figure 7**.

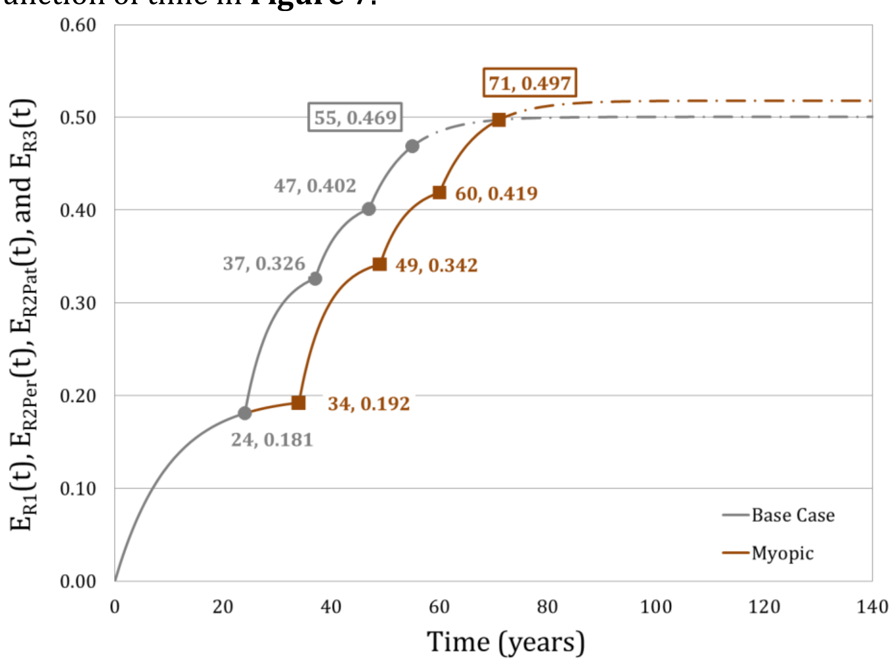

Figure 6: Recovery efficiency for myopic optimization and base case

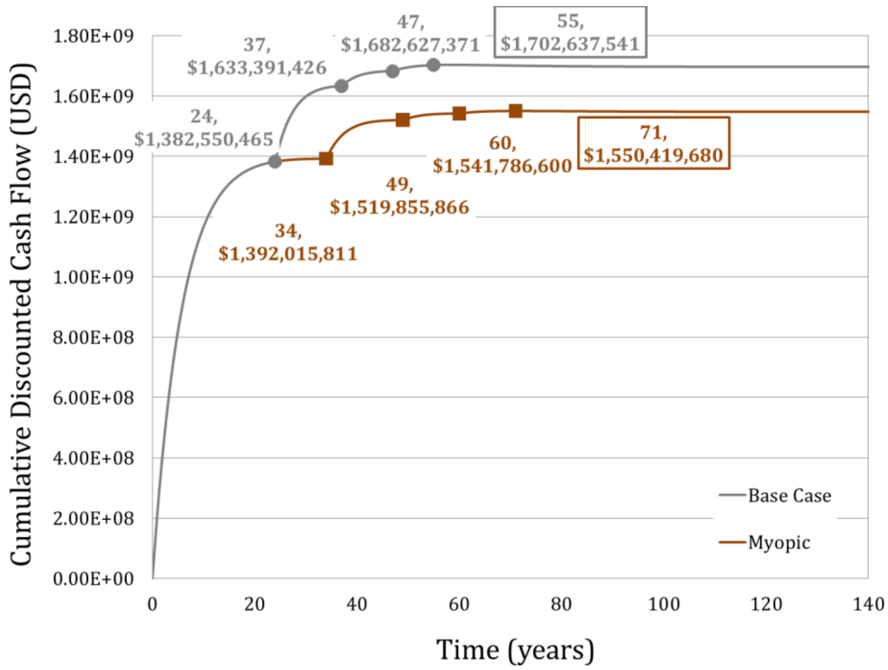

Figure 7: NPV myopic optimization and base case

Notice that the base case approaches a myopic optimization where the NPV is maximized per recovery phase.

2.6 Conclusion

This chapter sets the basis for the research. The successive analysis and optimization will be based on finding the values for the decision variables (t_{Life1}, t_{Life2} and t_{Life3}) that maximize the NPV assuming an exponential production profile. The optimization will be tackled in two ways: myopic or a short, medium-term approach, and life cycle or a long-term approach.

Chapter 3: Analytical Approach

3.1 Introduction

Chapter 2 provides the mathematics required to solve an optimization problem. The different equations offer enough detail to fit a real depletion profile, to graph recovery efficiency and NPV on a time scale, and to work with multiple assumptions in terms of costs, prices, interest rates, etc.

However, in practice, the finite series of cash flows (Equation 11) impose a serious limitation to the analysis. For every set of decision variables (t_{Life1}, t_{Life2} and t_{Life3}) the discounted stream of cash must be determined per previous period. For example, the assessment of the NPV for 30 years requires the sum of 30 discounted values. If expressed in months we would have 360 (30 years x 12 months/year) components and in days 10,950 (30 years x 365 days/year). For a Monte Carlo simulation of 1,000 trials we would need to find the optimal times per trial making the number of computations unmanageably large.

This chapter develops an analytical solution that allows obtaining the NPV for a set of decision variables in one equation without the need to discount the net cash flow per period. The importance of the analytical approach is significant when dealing with stochastic analysis where the numerical approach would not be viable.

Before we derive an analytical solution for the NPV we must introduce the concept of geometric series.

3.2 Geometric Series

A geometric series is the sum of the terms of a sequence with a constant ratio between successive terms. Each term is obtained from the preceding one by multiplying by a common factor known as geometric ratio.

$$a + ar + ar^3 + \cdots + ar^n = \sum_{n=0}^{\infty} ar^n$$

Equation 28

The geometric series is convergent if the absolute value of the geometric ratio is less than one, $|r| < 1$, and its sum is described as

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

Equation 29

for an infinite set of terms and

$$\sum_{n=0}^{N} ar^n = \frac{ar(1-r^N)}{1-r}$$

Equation 30

for an finite set of terms where

r = geometric ratio

We can apply now the geometric series for a finite set of terms to the model.

3.3 ANALYTICAL PHASE MODELING

3.3.1 Primary Recovery

As introduced in chapter 2, the NPV for primary production is given by

$$NPV_1 = \sum_{t=1}^{t_{Life1}} \frac{CF_{1t}}{(1+R)^t} = \sum_{t=1}^{t_{Life1}} \frac{Inflow_{1t} - Outflow_{1t}}{(1+R)^t}$$

or

$$NPV_1 = \sum_{t=1}^{t_{Life1}} \frac{N\$_{Oil}[E_{R1}(t) - E_{R1}(t-1)] - \left(\$_{Capex1} + \$_{Opex1}\right)}{(1+R)^t}$$

Equation 31

Note that starting at year 1 we are accounting for the cash flow generated from the first 12 months of production measured at the end of the year.

The incremental recovery efficiency for primary production at any time t can be written as

$$\mathrm{E_{R1}(t)-E_{R1}(t-1)}=\left[\mathrm{E_{R1}^{\infty}}\left(1-\mathrm{e}^{-\mathrm{t}/\tau_1}\right)\right]-\left[\mathrm{E_{R1}^{\infty}}\left(1-\mathrm{e}^{-(\mathrm{t}-1)/\tau_1}\right)\right]$$

and simplified

$$\mathrm{E_{R1}(t)-E_{R1}(t-1)}=\mathrm{E_{R1}^{\infty}}\left(\mathrm{e}^{-\mathrm{t}/\tau_1}\mathrm{e}^{1/\tau_1}-\mathrm{e}^{-\mathrm{t}/\tau_1}\right)=\mathrm{E_{R1}^{\infty}}\left(\mathrm{e}^{1/\tau_1}-1\right)\mathrm{e}^{-\mathrm{t}/\tau_1}$$

Equation 32

Substituting Equation 32 into the inflow term gives

$$Inflow_1=\sum_{t=1}^{t_{Life1}}\frac{N\$_{Oil}\mathrm{E_{R1}^{\infty}}\left(\mathrm{e}^{1/\tau_1}-1\right)\mathrm{e}^{-\mathrm{t}/\tau_1}}{(1+R)^t}=\sum_{t=1}^{t_{Life1}}N\$_{Oil}\mathrm{E_{R1}^{\infty}}\left(\mathrm{e}^{1/\tau_1}-1\right)\left(\frac{\mathrm{e}^{-1/\tau_1}}{1+R}\right)^t$$

Equation 33

where $a=N\$_{Oil}\mathrm{E_{R1}^{\infty}}\left(\mathrm{e}^{1/\tau_1}-1\right)$ and the geometric ratio $r=\frac{\mathrm{e}^{-1/\tau_1}}{(1+R)}$. Recall from Equation 30 that for a finite set of terms starting at n=1 we have

$$\sum_{n=1}^{N}ar^n=\frac{ar(1-r^N)}{1-r}$$

Equation 34

Therefore, we can eliminate the sum in Equation 33

$$Inflow_1=\frac{ar(1-r^N)}{1-r}=N\$_{Oil}\mathrm{E_{R1}^{\infty}}\left(\mathrm{e}^{1/\tau_1}-1\right)\left[\frac{\frac{\mathrm{e}^{-1/\tau_1}}{1+R}\left(1-\left(\frac{\mathrm{e}^{-1/\tau_1}}{1+R}\right)^{t_{Life1}}\right)}{1-\frac{\mathrm{e}^{-1/\tau_1}}{1+R}}\right]$$

Equation 35

Multiplying the numerator and denominator by (1+R) gives

$$Inflow_1 = N\$_{Oil}\mathrm{E}_{\mathrm{R1}}^{\infty}\left(\mathrm{e}^{1/\tau_1} - 1\right)\left[\frac{\mathrm{e}^{-1/\tau_1}\left(1 - \left(\frac{\mathrm{e}^{-1/\tau_1}}{1+R}\right)^{t_{Life1}}\right)}{1 + \mathrm{R} - \mathrm{e}^{-1/\tau_1}}\right]$$

Equation 36

and rearranging terms results in

$$Inflow_1 = N\$_{Oil}\mathrm{E}_{\mathrm{R1}}^{\infty}\left(1 - \mathrm{e}^{-1/\tau_1}\right)\left[\frac{1 - \left(\frac{\mathrm{e}^{-1/\tau_1}}{1+R}\right)^{t_{Life1}}}{1 + \mathrm{R} - \mathrm{e}^{-1/\tau_1}}\right]$$

Equation 37

Similarly, to derive the outflow or costs associated with primary production we follow the same procedure as before. Starting from

$$Outflow_1 = -\sum_{t=1}^{t_{Life1}} \frac{\$_{Capex1} + \$_{Opex1}}{(1+R)^t}$$

Equation 38

and assuming that the capital expenditure is amortized at early stages (t=1) of the project gives

$$Outflow_1 = -\$_{Capex1} - \sum_{t=1}^{t_{Life1}} \frac{\$_{Opex1}}{(1+R)^t}$$

Equation 39

where $a = \$_{Opex1}$ and the geometric ratio $r = 1/(1+R)$. Using Equation 34 we eliminate the sum and obtain

$$Outflow_1 = -\$_{Capex1} - \$_{Opex1}\left[\frac{\frac{1}{1+R}\left(1 - \left(\frac{1}{1+R}\right)^{t_{Life1}}\right)}{1 - \frac{1}{1+R}}\right]$$

Equation 40

Multiplying the numerator and denominator by (1+r) results in

$$Outflow_1 = -\$_{Capex1} - \frac{\$_{Opex1}}{R}\left[1 - \left(\frac{1}{1+R}\right)^{t_{Life1}}\right]$$

Equation 41

Combining Equation 37 and Equation 41 we have the analytical expression for the NPV generated by a reservoir undergoing primary production over a life of $t = t_{Life1}$.

$$NPV_1 = N\$_{oil}E_{R1}^{\infty}\left(1 - e^{-\frac{1}{\tau_1}}\right)\left[\frac{1 - \left(\frac{e^{-\frac{1}{\tau_1}}}{1+R}\right)^{t_{Life1}}}{1 + R - e^{-\frac{1}{\tau_1}}}\right] - \$_{Capex1}$$
$$- \frac{\$_{Opex1}}{R}\left[1 - \left(\frac{1}{1+R}\right)^{t_{Life1}}\right]$$

Equation 42

3.3.2 Secondary Recovery

From Equation 4 the recovery efficiency for secondary production is

$$E_{R2}(t) = E_{R1}(t_{Life1}) + \left[E_{R2}^{\infty\prime} - E_{R1}(t_{Life1})\right]\left(1 - e^{-(t - t_{Life1})/\tau_2}\right) \; for \; t \geq t_{Life1}$$

where

$$E_{R1}(t_{Life1}) = E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right)$$

$$E_{R2}^{\infty\prime} = E_{R1}(t_{Life1}) + \Delta E_{R2}^{\infty} = E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right) + \left(E_{R2}^{\infty} - E_{R1}^{\infty}\right)$$

Inserting both expressions into Equation 4 we obtain

$$E_{R2}(t) = E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right) + \left[E_{R2}^{\infty\prime} - E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right)\right]\left(1 - e^{-(t - t_{Life1})/\tau_2}\right)$$

and for $t - 1$

$$E_{R2}(t-1) = E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right) + \left[E_{R2}^{\infty\prime} - E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right)\right]\left(1 - e^{-(t - t_{Life1} - 1)/\tau_2}\right)$$

The incremental recovery can then be expressed as

$$E_{R2}(t) - E_{R2}(t-1) = \left[E_{R2}^{\infty\prime} - E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right)\right]\left(e^{-(t - t_{Life1} - 1)/\tau_2} - e^{-(t - t_{Life1})/\tau_2}\right)$$

Equation 43

Rearranging the terms to place it into a geometric series notation gives

$$E_{R2}(t) - E_{R2}(t-1) = \left[E_{R2}^{\infty\prime} - E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right)\right]\left(e^{t_{Life1} - 1/\tau_2} - e^{t_{Life1}/\tau_2}\right)e^{-t/\tau_2}$$

Equation 44

Recall from Equation 19 and Equation 20 that the NPV for secondary recovery is

$$NPV_2 = \sum_{t_{Life1}}^{t_{Life2}} \frac{CF_{2t}}{(1+R)^t} = \frac{N\$_{Oil}[E_{R2}(t) - E_{R2}(t-1)] - \left(\$_{Capex2} + \$_{Opex2}\right)}{(1+R)^t}$$

Substituting Equation 44 into the inflow term gives

$$Inflow_2 = \sum_{t_{Life1}}^{t_{Life2}} \frac{N\$_{Oil}\left[E_{R2}^{\infty\prime} - E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right)\right]\left(e^{t_{Life1} - 1/\tau_2} - e^{t_{Life1}/\tau_2}\right)e^{-t/\tau_2}}{(1+R)^t}$$

or

$$Inflow_2 = \sum_{t_{Life1}}^{t_{Life2}} N\$_{Oil}\left[E_{R2}^{\infty\prime} - E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right)\right]\left(e^{t_{Life1} - 1/\tau_2} - e^{t_{Life1}/\tau_2}\right)\left(\frac{\mathrm{e}^{-1/\tau_2}}{1+R}\right)^t$$

Equation 45

where $a = N\$_{Oil}\left[E_{R2}^{\infty\prime} - E_{R1}^{\infty}\left(1 - e^{-t_{Life1}/\tau_1}\right)\right]\left(e^{t_{Life1} - 1/\tau_2} - e^{t_{Life1}/\tau_2}\right)$ and the geometric ratio $r = \left(\mathrm{e}^{-1/\tau_2}\right)/(1+R)$.

Using Equation 34 to eliminate the sum, multiplying the numerator and denominator by (1+R) and rearranging results in

$$NPV_2 = N\$_{oil}\left\{\frac{\left[E_{R2}^{\infty\prime} - E_{R1}^{\infty}\left(1 - e^{-\frac{t_{Life1}}{\tau_1}}\right)\right]\left(e^{\frac{t_{Life1}}{\tau_2}} - e^{\frac{t_{Life1}-1}{\tau_2}}\right)\left[\left(\frac{e^{-\frac{1}{\tau_2}}}{1+R}\right)^{t_{Life1}} - \left(\frac{e^{-\frac{1}{\tau_2}}}{1+R}\right)^{t_{Life1}+t_{Life2}}\right]}{1 + R - e^{-\frac{1}{\tau_2}}}\right\}$$

Equation 46

Combining the outflow and the inflow terms as in primary recovery we have the analytical expression for the contribution to the NPV of secondary recovery.

$$NPV_2 = N\$_{oil}\left\{\frac{\left[E_{R2}^{\infty\prime} - E_{R1}^{\infty}\left(1 - e^{-\frac{t_{Life1}}{\tau_1}}\right)\right]\left(e^{\frac{t_{Life1}}{\tau_2}} - e^{\frac{t_{Life1}-1}{\tau_2}}\right)\left[\left(\frac{e^{-\frac{1}{\tau_2}}}{1+R}\right)^{t_{Life1}} - \left(\frac{e^{-\frac{1}{\tau_2}}}{1+R}\right)^{t_{Life1}+t_{Life2}}\right]}{1 + R - e^{-\frac{1}{\tau_2}}}\right\} - \frac{\$_{Capex2}}{(1+R)^{t_{Life1}}} - \frac{\$_{Opex2}}{R}\left(\frac{1}{(1+R)^{t_{Life1}}} - \frac{1}{(1+R)^{t_{Life1}+t_{Life2}}}\right)$$

Equation 47[3]

Note Equation 47 assumes capital expenditure for secondary production to be paid in full at the end of primary production and before secondary production starts.

[3] For the two phase problem there is only one decision variable. The solution can be optimized analytically finding a closed-form formula for the optimal switching time.

3.3.3 Tertiary Recovery

Following the same manipulations as in primary and secondary recovery, we derive the contribution to the NPV of tertiary recovery (Equation 48).

$$N\$_{oil}\left\{\frac{\left\{E_{R_3}^{\infty\prime}-\left[E_{R_2}^{\infty\prime}-\left(E_{R_2}^{\infty\prime}-\left[E_{R_1}^{\infty}\left(1-e^{-\frac{t_{Life1}}{\tau_1}}\right)\right]\left(e^{\frac{-t_{Life2}}{\tau_2}}\right)\right)\right]\right\}\left(e^{\left(\frac{t_{Life2}+t_{Life1}}{\tau_3}\right)}-e^{\left(\frac{t_{Life2}+t_{Life1}-1}{\tau_3}\right)}\right)\left[\left(\frac{e^{-\frac{1}{\tau_3}}}{1+R}\right)^{t_{Life1}+t_{Life2}}-\left(\frac{e^{-\frac{1}{\tau_3}}}{1+R}\right)^{t_{Life1}+t_{Life2}+t_{Life3}}\right]}{1+R-e^{-\frac{1}{\tau_3}}}\right\}$$

$$-\frac{\$_{Capex3}}{(1+R)^{t_{Life1}+t_{Life2}}}-\frac{\$_{Opex3}}{R}\left(\frac{1}{(1+R)^{t_{Life1}+t_{Life2}+t_{Life3}}}-\frac{1}{(1+R)^{t_{Life1}+t_{Life2}}}\right)=NPV_3$$

Equation 48

The NPV for the entire project is given by

$$NPV_{Total}=\sum_{i=1}^{n}NPV_i=NPV_1+NPV_2+NPV_3$$

or

$$
\begin{aligned}
&NPV_{Total} = \\
&= N\$_{oil} E_{R1}^{\infty}\left(1 - e^{-\frac{1}{\tau_1}}\right)\left[\frac{1 - \left(\frac{e^{-\frac{1}{\tau_1}}}{1+R}\right)^{t_{Life1}}}{1 + R - e^{-\frac{1}{\tau_1}}}\right] - \$_{Capex1} - \frac{\$_{Opex1}}{R}\left[1 - \left(\frac{1}{1+R}\right)^{t_{Life1}}\right] \\
&+ N\$_{oil}\left\{\frac{\left[E_{R2}^{\infty'} - E_{R1}^{\infty}\left(1 - e^{-\frac{t_{Life1}}{\tau_1}}\right)\right]\left(e^{\frac{t_{Life1}}{\tau_2}} - e^{\frac{t_{Life1}-1}{\tau_2}}\right)\left[\left(\frac{e^{-\frac{1}{\tau_2}}}{1+R}\right)^{t_{Life1}} - \left(\frac{e^{-\frac{1}{\tau_2}}}{1+R}\right)^{t_{Life1}+t_{Life2}}\right]}{1 + R - e^{-\frac{1}{\tau_2}}}\right\} - \frac{\$_{Capex2}}{(1+R)^{t_{Life1}}} \\
&- \frac{\$_{Opex2}}{R}\left(\frac{1}{(1+R)^{t_{Life1}}} - \frac{1}{(1+R)^{t_{Life1}+t_{Life2}}}\right) \\
&+ N\$_{oil}\left\{\frac{\left\{E_{R_3}^{\infty'} - \left[E_{R_2}^{\infty'} - \left(E_{R_2}^{\infty'} - \left[E_{R_1}^{\infty}\left(1 - e^{-\frac{t_{Life1}}{\tau_1}}\right)\right]\left(e^{\frac{-t_{Life2}}{\tau_2}}\right)\right)\right]\right\}\left(e^{\left(\frac{t_{Life2}+t_{Life1}}{\tau_3}\right)} - e^{\left(\frac{t_{Life2}+t_{Life1}-1}{\tau_3}\right)}\right)\left[\left(\frac{e^{-\frac{1}{\tau_3}}}{1+R}\right)^{t_{Life1}+t_{Life2}} - \left(\frac{e^{-\frac{1}{\tau_3}}}{1+R}\right)^{t_{Life1}+t_{Life2}+t_{Life3}}\right]}{1 + R - e^{-\frac{1}{\tau_3}}}\right\} \\
&- \frac{\$_{Capex3}}{(1+R)^{t_{Life1}+t_{Life2}}} - \frac{\$_{Opex3}}{R}\left(\frac{1}{(1+R)^{t_{Life1}+t_{Life2}+t_{Life3}}} - \frac{1}{(1+R)^{t_{Life1}+t_{Life2}}}\right)
\end{aligned}
$$

Equation 49[4]

[4] Analytical solution derived by David Livasy, 2010.

3.4 ILLUSTRATIVE EXAMPLE

3.4.1 Problem Statement and Assumptions

Using the same field and assumptions as in chapter's 2 illustrative example:

a. Maximize the NPV myopically (per recovery phase). Compute the total NPV for this myopic approach. What would be the values for t_{Life1}, t_{Life2} and t_{Life3}?

b. Determine the total NPV according to **Figure 4** production profile (base case).

c. Compare the results from the numerical and analytical approaches.

3.4.2 Solution to Illustrative Example

a. Maximize the NPV per recovery phase. Compute the total NPV for this myopic approach. What would be the values for t_{Life1}, t_{Life2} and t_{Life3}?

The NPV is given by Equation 49 where NPV_2 corresponds to $NPV_{2Peripheral}$ and NPV_3 to $NPV_{2Pattern}$. Equation 50 includes the additional term derive for the CO_2 contribution to the NPV. This term must be added to Equation 49 to compute the total NPV.

Table 13 shows the values of the components that are input into the NPV equation. Refer to solution to illustrative example in chapter 2 for a detail explanation on how to obtain intermediate results.

Table 13: NPV per recovery phase calculated analytically (myopic optimization)

	PRIMARY	SECONDARY PER	SECONDARY PATT	TERTIARY
t_{Lifei} (years)	34	15	11	11
$Inflow_i$	$1,508,434,874	$147,130,078	$28,509,336	$13,321,250
$Outflow_i$	-$116,419,063	-$19,290,023	-$6,578,602	-$4,688,170
NPV_i	**$1,392,015,811**	**$127,840,055**	**$21,930,734**	**$8,633,080**

$$
\begin{aligned}
NPV_{CO2} &= NPV_3 \\
&= N\$_{oil}\left\{\frac{\left\{E_{R3}^{\infty'}-\left(E_{2Pat}^{\infty'}-\left\{E_{2Pat}^{\infty'}-\left[E_{R2Per}^{\infty'}-\left(E_{R2Per}^{\infty'}-\left[E_{R1}^{\infty}\left(1-e^{-\frac{t_1}{\tau_1}}\right)\right]\left(e^{\frac{-t_2}{\tau_{2Per}}}\right)\right)\right]\right\}\left(e^{-\frac{t_2}{\tau_{2Pat}}}\right)\right)\right\}\left(e^{\left(\frac{t_1+t_2+t_{2'}}{\tau_3}\right)}-e^{\left(\frac{t_1+t_2+t_{2'}-1}{\tau_3}\right)}\right)\left[\left(\frac{e^{-\frac{1}{\tau_3}}}{1+R}\right)^{t_1+t_2+t_{2'}}-\left(\frac{e^{-\frac{1}{\tau_3}}}{1+R}\right)^{t_1+t_2+t_{2'}+t_3}\right]}{1+R-e^{-\frac{1}{\tau_3}}}\right\} \\
&\quad -\frac{\$_{Capex3}}{(1+R)^{t_{Life1}+t_{Life2Per}+t_{Life2Pat}}}-\frac{\$_{Opex3}}{R}\left(\frac{1}{(1+R)^{t_{Life1}+t_{Life2Per}+t_{Life2Pat}+t_{Life3}}}-\frac{1}{(1+R)^{t_{Life1}+t_{Life2Per}+t_{Life2Pat}}}\right)
\end{aligned}
$$

Equation 50[5]

[5] Due to lack of space in the first term of the equation t_{Life1} has been abbreviated by t_1, $t_{Life2Peripheral}$ by t_2, $t_{Life2Pattern}$ by $t_{2'}$ and t_{Life3} by t_3.

The total NPV is equal to the sum of the NPV from each production stage.

$$NPV_{Total} = \sum_{i=1}^{n} NPV_i = NPV_1 + NPV_{2Peripheral} + NPV_{2Pattern} + NPV_3$$

$$= \$1{,}392{,}015{,}811 + \$127{,}840{,}055 + \$21{,}930{,}734 + \$8{,}633{,}080$$

$$= \$1{,}550{,}419{,}680$$

To determine the maximum NPV per recovery phase using the analytical solution we employ Excel Solver®. For a myopic approach the optimization in Solver must be sequential. We first maximize the NPV subject to changes in t_1 to obtain t_{Life1}. Then we maximize the NPV by changing $t_{Life2Peripheral}$ to obtain $t_{Life2Peripheral}$ and proceed in a similar fashion for $t_{Life2Pattern}$ and t_{Life3}.

b. Determine the total NPV according to **Figure 4** production profile (base case).

For the base case t_{Life1}, $t_{Life2Peripheral}$, $t_{Life2Pattern}$ and t_{Life3} are obtained from **Figure 4**. **Table 14** shows the values of the different components that are input into the total NPV equation and the NPV for the base case.

Table 14: NPV per recovery phase calculated analytically (base case)

	PRIMARY	SECONDARY PER	SECONDARY PATT	TERTIARY
t_{Lifei} (years)	24	13	10	8
$Inflow_i$	$1,486,428,491	$285,661,528	$63,113,504	$29,006,735
$Outflow_i$	-$103,878,026	-$34,820,566	-$13,877,559	-$8,996,565
NPV_i	**$1,382,550,465**	**$250,840,961**	**$49,235,945**	**$20,010,170**

$$NPV_{Total} = \sum_{i=1}^{n} NPV_i = NPV_1 + NPV_{2Peripheral} + NPV_{2Pattern} + NPV_3$$

$$= \$1{,}382{,}550{,}465 + \$250{,}840{,}961 + \$49{,}235{,}945 + \$20{,}010{,}170$$

$$= \$1{,}702{,}637{,}541$$

c. Compare the results from the numerical and analytical approaches.

The illustrative example in chapters 2 and 3 proves that numerical and analytical approaches can be used independently leading to the same solutions (**Table 15**). The only sources of discrepancies between the two approaches were observed when the optimal values of t_{Lifei} were not integers. This is because the discretization of the time in the numerical model where the present values are computed per year. **Table 16** provides a comparison between the two methods.

Table 15: Results from numerical and analytical approaches

	Myopic		Base Case	
	PRIMARY		PRIMARY	
	Numerical	**Analytical**	**Numerical**	**Analytical**
NPV_1	\$1,392,015,811	\$1,392,015,811	\$1,382,550,465	\$1,382,550,465
Error	**0%**		**0%**	
	SECONDARY PERIPHERAL		SECONDARY PERIPHERAL	
	Numerical	**Analytical**	**Numerical**	**Analytical**
NPV_{2Pat}	\$127,840,055	\$127,840,055	\$250,840,961	\$250,840,961
NPV_1+NPV_2	\$1,519,855,866	\$1,519,855,866	\$1,633,391,426	\$1,633,391,426
Error	**0%**		**0%**	
	SECONDARY PATTERN		SECONDARY PATTERN	
	Numerical	**Analytical**	**Numerical**	**Analytical**
NPV_{2Per}	\$21,930,734	\$21,930,734	\$49,235,945	\$49,235,945
NPV_1+NPV_2+NPV_2	\$1,541,786,600	\$1,541,786,600	\$1,682,627,371	\$1,682,627,371
Error	**0%**		**0%**	
	TERTIARY		TERTIARY	
	Numerical	**Analytical**	**Numerical**	**Analytical**
NPV_3	\$8,633,080	\$8,633,080	\$20,010,170	\$20,010,170
NPV_1+NPV_2+NPV_3	\$1,550,419,680	\$1,550,419,680	\$1,702,637,541	\$1,702,637,541
Error	**0%**		**0%**	

Table 16: Numerical versus analytical approach

	NUMERICAL	ANALYTICAL
ADVANTAGES	- Required for data fit - NPV can be determined for R=0 - NPV and recovery efficiency can be plotted as a function of time	- Fastest approach for myopic optimization - Virtually the only option for life cycle optimization when R≠0
DISADVANTAGES	- Number of computations can become unmanageably large - Limited use for life cycle optimations when R≠0	- Cannot be used for data fit - Cannot be used when R=0 - Limited use for plotting results

3.5 CONCLUSION

Part A contains the mathematical tools used to build the model for this thesis. We will refer to these formulae throughout the document.

The numerical and analytical equations developed in chapters 2 and 3 can be used independently of each other. In general we employ the analytical approach to run the optimizations in both the deterministic and the stochastic evaluation; and the numerical approach to fit production data and plot the recovery efficiency and NPV results.

The next chapter delves into the illustrative example introduce in chapters 2 and 3.

PART B: DETERMINISTIC EVALUATION OF THE MODEL

In part B we test the method presented in chapters 2 and 3 with a deterministic evaluation of the model. The deterministic approach is applied to the illustrative example introduced in chapter 2.

Chapter 4: Base Case

4.1 INTRODUCTION

Figure 8 shows the recovery efficiency as a function of time for the field in south central Wyoming presented previously and **Table 17** summarizes the reservoir parameters extracted from this figure.

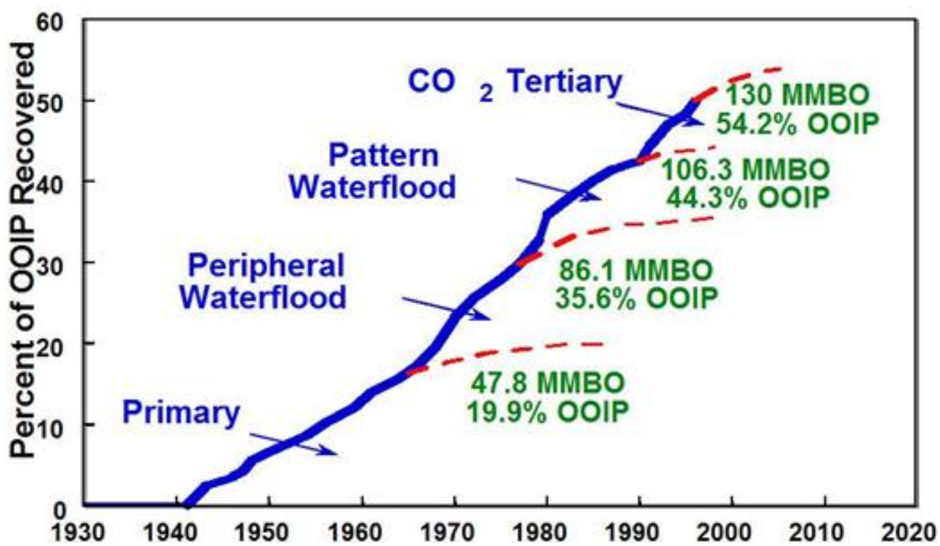

Figure 8: Recovery efficiency as a function of time. From Brokmeyer et al., 1996

Table 17: Base case reservoir parameters

Parameters	Values
E_{R1}^{∞} (fraction)	0.199
τ_1 (years)	10
ΔE_{R2}^{∞} (fraction)	0.157
τ_2 (years)	5
ΔE_{R2}^{∞} (fraction)	0.087
τ_2 (years)	5
ΔE_{R3}^{∞} (fraction)	0.099
τ_3 (years)	7
N	240,000,000

Similarly, **Table 18** displays the values assumed for the illustrative example (refer to section 2.5.3 for further detail on estimating yearly operating cost).

Table 18: Yearly operating cost per recovery phase

Recovery Phase$_i$	Opex ($/bbl)	$\$_{Opexi}$
1. Primary	3	$9,057,023
2. Secondary Peripheral	7	$21,133,055
3. Secondary Pattern	8	$24,152,062
4. Tertiary	12	$36,228,093

We assume the oil price is $55 per barrel and the discount rate in real terms is 7%/year.

4.2 OPTIMIZATION METHOD

Throughout the research we approach optimization in two different ways: myopically and life cycle optimization. The analysis in this chapter includes as well the base case. The next three sections review the three optimizations and the procedure used.

4.2.1 Myopic Optimization

Historically, industry practices have attempted to maximize the NPV with short to medium–term strategies that focus, in the majority of cases, on a 2-5 year window. To reproduce this strategy we maximize the NPV per recovery phase in the myopic approach using the analytical solution derived in Equations 49 and 50 and Excel Solver®. For a myopic approach the optimization in Solver must be sequential. First we maximize the NPV subject to changes in t_1 to obtain t_{Life1}. Then we maximize the NPV by changing $t_{2Peripheral}$ to obtain $t_{Life2Peripheral}$ and proceed in a similar fashion for $t_{Life2Pattern}$ and t_{Life3}.

4.2.2 Base Case Optimization

From **Figure 8** we estimate primary production for the Base Case to last 24 years. Peripheral waterflooding follows for 13 years, pattern waterflooding for 10 years, and CO_2 for 8 years. We take these values to be the base case optimization. The NPV values can be computed numerically or analytically.

4.2.3 Life Cycle Optimization

A different approach to a myopic optimization, probably the opposite, would be a long-term strategy or a life cycle optimization. The aim of a life cycle optimization is to determine the time that must be devoted to each recovery phase to maximize the NPV of the project over its life.

We achieve a life cycle optimization in Excel Solver® by finding the values for the decision variables (t_{Life1}, t_{Life2}, and t_{Life3}) that maximize the NPV per recovery phase. Unlike the myopic approach, the optimization of the decision variables is done simultaneously to obtain a life cycle solution.

The NPV is determined using the analytical solution derived in Equations 49 and 50. Although both myopic and life cycle optimization are solved in Excel Solver® from the analytical solution, the NPV and recovery efficiencies can only be plotted using the numerical approach (refer to illustrative example in chapters 2 and 3 for a comparison between numerical and analytical approach).

4.3 COMPARING MYOPIC, BASE CASE AND LIFE CYCLE RESULTS

4.3.1 Net Present Value

Table 19: Myopic, base case and life cycle results

	Myopic Optimization	Base Case Optimization	Life Cycle Optimization
NPV ($bn)	**$1.55**	**$1.70**	**$2.06**
t_{Life} (years)	**71**	**55**	**34**
Cummulative Recovery Efficiency	**0.497**	**0.469**	**0.379**

Table 19 summarizes the results for the three cases. The NPV for the life cycle optimization is 21% larger than obtained for the base case and 33% larger than the myopic optimization. The life of the project is drastically reduced when maximizing the NPV globally. This reduction accounts for 21 years for the base case and 37 years for the myopic optimization. Finally, the percentage of oil recovered decreases from 49.73% for the myopic optimization and 46.89% for the base case to 37.22% for the life cycle solution.

Figure 9 presents the cumulative discounted cash flow (CDCF) as a function of time. The CDCF in dollars is graphed on the y axis and the time in years is on the x axis.

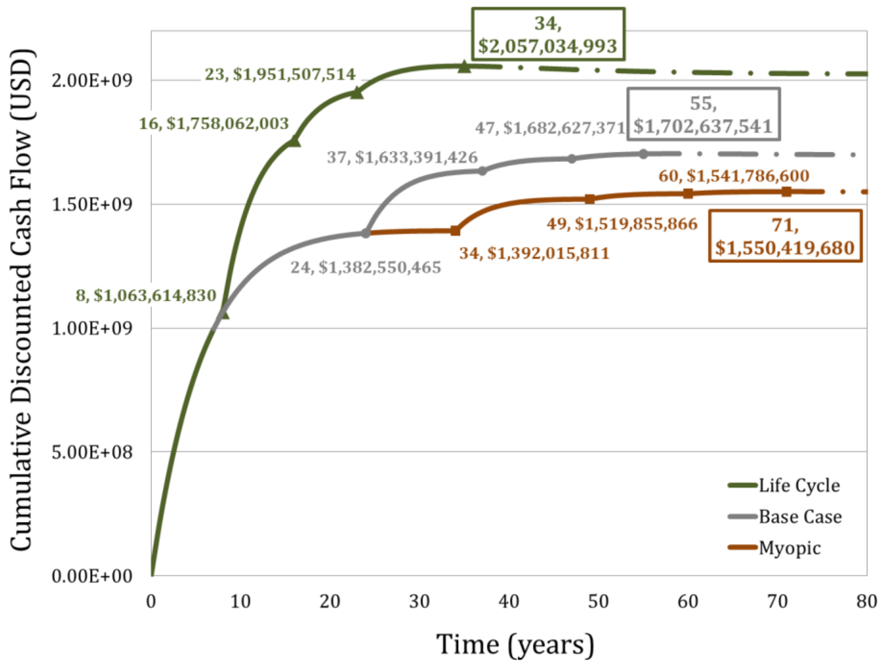

Figure 9: Cumulative discounted cash flow as a function of time

For the life cycle optimization, the contribution to the NPV of primary, peripheral waterflooding, pattern waterflooding and CO_2 is 52%, 34%, 9% and 5%, respectively. The difference between primary, secondary and tertiary contribution to the NPV becomes more apparent for the base case and the myopic optimization. For the base case primary production accounts for 81% of the total NPV and CO_2 only adds 1% to the project. For myopic optimization primary takes 90% of the NPV and tertiary is only 1%.

The time value of money is taken into account when discounting the cash flow. As the decision of implementing a new recovery method is delayed into the future, its discounted value decreases. This effect is observed in the base case and is even more evident for the myopic optimization.

4.3.2 Recovery Efficiency

Figure 10 shows the recovery efficiency as a function of time. Recovery efficiency is plotted on the y axis and time in years on the x axis.

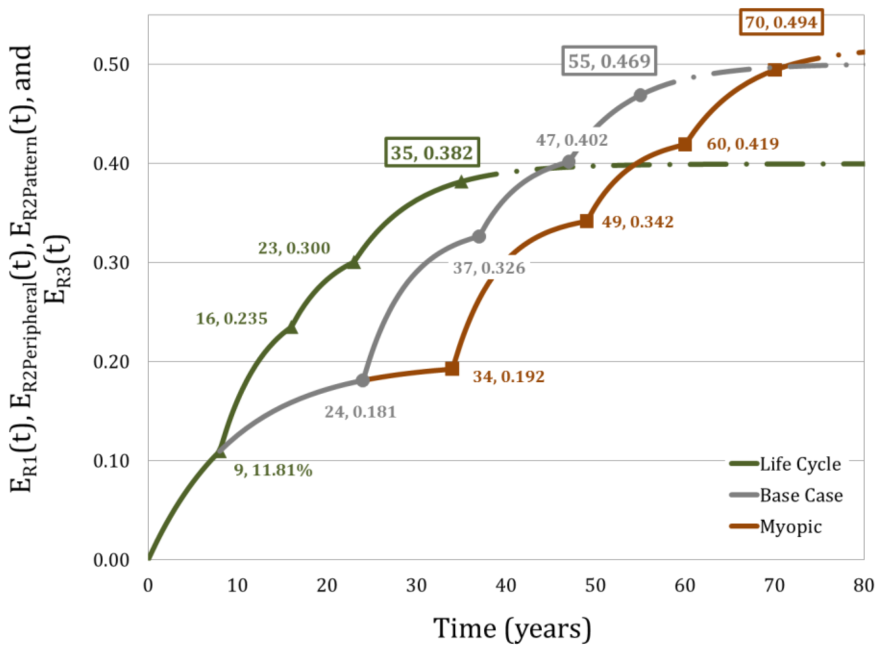

Figure 10: Recovery efficiency as a function of time

We observe that the incremental recovery efficiency of CO_2 is 7.84% for both myopic and life cycle optimization. Indeed, both strategies are equivalent for tertiary recovery. Either with a myopic or a life cycle approach, production continues until the economic limit is reached (when injecting CO_2). The reduction on the percentage of oil recovered with the life cycle optimization is explained by the smaller recovery efficiency obtained from primary production and to a lesser degree from waterflooding. After 7 years (or optimal time assigned to primary recovery using a life cycle optimization), recovery efficiency and NPV compete with each other. Additional production increases recovery efficiency to the detriment of the net present value and vice versa. This becomes clear when we compare the results of the life cycle and myopic optimization. The first optimization produces an NPV 33% larger than the second optimization. On the other hand, the myopic solution results in an additional 11% of oil recovered.

It is important to emphasize that a life cycle optimization provides the optimal production schedule for a given cost structure and discount rate. The life of the project for the life cycle optimization, in this case 34 years, indicates the time in which the reservoir should be abandoned to maximize the NPV given a specific cost structure. A different cost of capital or another operator specialized in depleted fields could provide a more efficient operation that justifies further production while increasing the NPV.

4.3.3 Life of the Project

Figure 10 illustrates the time assigned to each recovery phase for the life cycle, base case and myopic optimization. We observe that the larger discrepancies correspond to primary recovery. The differences decrease for secondary production and for tertiary recovery the life cycle and myopic approach show the same results. As mentioned before, when there are no more recovery mechanisms available, production continues until the economic limit is reached. It is reasonable to believe that the base case is being produced above 8 years although the data that we have from that field stops at that point (year 1998).

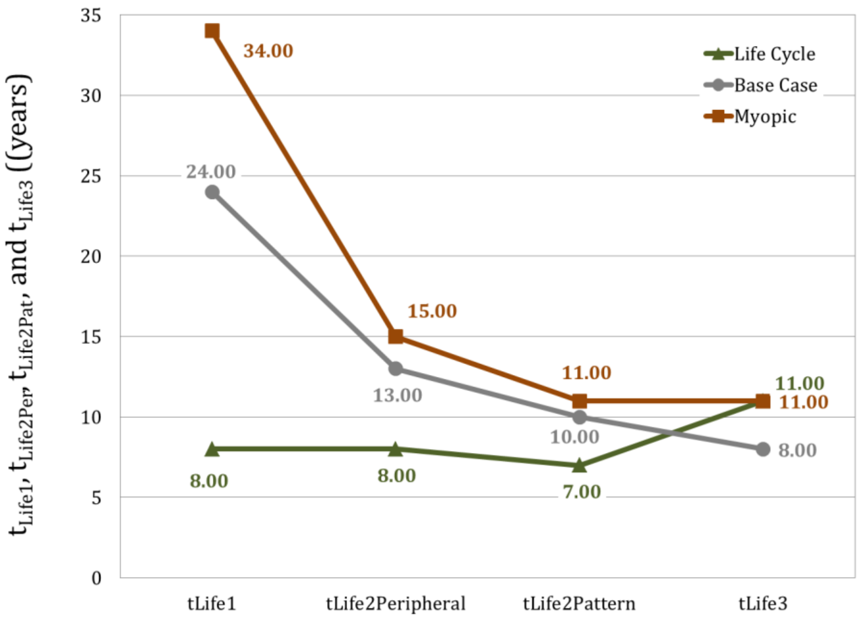

Figure 11: t_{Lifei} per recovery phase

4.4 SENSITIVITY ANALYSIS

Sensitivity analysis is used to determine how sensitive the output of the model is to changes in the value of the parameters of the model. Parameter sensibility is usually performed by setting different parameter values and evaluating how those changes in the parameters affect the net present value and the decision variables of the model. Sensitivity analysis helps to test the robustness of the model by studying the uncertainties that are often associated with parameters that are relevant but we do not have control over them and are difficult to predict.

The net present value is affected by technical and economic uncertainties. These uncertainties compromise the ability to make adequate decisions. Therefore, we want to analyze not only how sensitive the NPV is to changes in the different parameters but also how that affect our decision variables (t_{Lifei}).

The sensibility analysis is performed over the life cycle optimization for the base case. We conduct sensibility analysis in all the constant parameters in the model.

4.4.1 Method 1: Variable Ranging

In variable ranging one parameter is multiplied by a scaling factor while the other parameters remain constant. We used a scaling factor of ±50% of the original value and run Excel Solver® to maximize the NPV subject to an updated exploitation strategy.

The results are separated between reservoir and economic uncertainties. The scaling factor is applied according to the layout in **Table 20**.

Table 20: Scaling factor

Parameter	Scaling factor
Lower Limit	50%
Base Case	100%
Higher Limit	150%

4.4.1.1Sensitivity analysis on reservoir parameters

Table 21 summarizes the optimized results for the sensitivity analysis on the reservoir parameters. The first three rows show the effect of changing theoretical ultimate recovery for primary production while fixing the other parameters. In the next three rows the time constant for production is changed, we continue with peripheral waterflooding parameters, pattern waterflooding and end with tertiary recovery.

Table 21: Sensibility analysis on reservoir parameters

Parameter	Value	NPV % change	NPV ($bn)	t_{Life1} (years)	t_{Life2} (years)	$t_{Life2Pat}$ (years)	t_{Life3} (years)	t_{Life} (years)	$ER_3(t)$
E_{R1}^{∞}	10.0%	-17%	$1.71	0.21	8.04	6.77	11.47	15.02	0.272
E_{R1}^{∞}	**19.9%**	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
E_{R1}^{∞}	29.9%	29%	$2.65	11.19	8.04	6.77	11.47	26.01	0.471
τ_1	5	27%	$2.62	7.03	8.04	6.77	11.47	21.85	0.420
τ_1	**10**	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
τ_1	15	-13%	$1.80	4.63	8.04	6.77	11.47	19.44	0.323
$\Delta E_{R2Per}^{\infty}$	7.85%	-16%	$1.73	11.19	4.57	6.77	11.47	22.54	0.325
$\Delta E_{R2Per}^{\infty}$	**15.70%**	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
$\Delta E_{R2Per}^{\infty}$	23.55%	23%	$2.53	3.98	10.07	6.77	11.47	20.82	0.414
τ_{2Per}	2.5	12%	$2.32	5.23	5.75	6.77	11.47	17.75	0.367
τ_{2Per}	**5**	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
τ_{2Per}	7.5	-7%	$1.91	8.63	9.02	6.77	11.47	24.42	0.369
$\Delta E_{R2Pat}^{\infty}$	4.4%	-5%	$1.96	8.12	9.55	3.31	11.47	20.97	0.345
$\Delta E_{R2Pat}^{\infty}$	**8.7%**	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
$\Delta E_{R2Pat}^{\infty}$	13.1%	7%	$2.20	6.00	6.70	8.80	11.47	21.51	0.394
τ_{2Pat}	2.5	4%	$2.15	6.41	7.15	5.12	11.47	18.68	0.369
τ_{2Pat}	**5**	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
τ_{2Pat}	7.5	-2%	$2.01	7.60	8.70	7.12	11.47	23.42	0.368
ΔE_{R3}^{∞}	4.95%	-3%	$1.99	7.81	9.03	9.60	6.62	26.44	0.344
ΔE_{R3}^{∞}	**9.90%**	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
ΔE_{R3}^{∞}	14.85%	5%	$2.16	6.29	7.01	4.84	14.31	18.13	0.394
τ_3	3.5	3%	$2.12	6.64	7.40	5.51	8.16	19.55	0.365
τ_3	**7**	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
τ_3	10.5	-2%	$2.03	7.44	8.45	7.78	12.95	23.67	0.371
N (bbls)	120,000,000	-55%	$0.92	8.02	7.91	6.14	6.62	22.07	0.357
N (bbls)	**240,000,000**	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
N (bbls)	360,000,000	56%	$3.21	6.82	8.02	6.86	14.31	21.70	0.375

Note: Base case in bold

From **Table 21** we can infer the following results:

- Changes in recovery efficiency and time constant for primary production have a large effect on the NPV. A 50% increase on the recovery efficiency forecast has an impact on NPV similar to 50% reduction on the time constant for production for primary recovery.

- The importance of peripheral waterflooding is also significant; however, recovery efficiency in this case is more critical than the time constant for production.
- Pattern waterflooding and tertiary recovery have a limited effect over the NPV. Recall from the base case analysis that the contribution of pattern waterflooding and CO_2 to the NPV was relatively small compared to primary and peripheral waterflooding because the discounting effect.
- The exploitation strategy is largely affected by these changes. As it is the case with the NPV, the impact is larger for primary and peripheral waterflooding than it is for pattern waterflooding and CO_2.
- The original oil in place (OOIP) shows the highest impact over the NPV of all the reservoir parameters. This effect is expected if we look at the equations derived in the numerical and analytical approach. The OOIP and the oil price (analyze in the next section) multiply every inflow term of the NPV.
- The effect of changes on the OOIP over the exploitation strategy is marginal.

4.4.1.2Sensitivity analysis on economic parameters

Table 22 summarizes the optimized results for the sensitivity analysis on the economic parameters. The first three rows show the effect of changing the discount rate in real terms while fixing the other parameters, in the next three rows the oil price varies. We finish with the operating costs per recovery phase.

Table 22: Sensibility analysis on economic parameters

Parameter	Value	NPV % change	NPV ($bn)	t_{Life1} (years)	t_{Life2} (years)	$t_{Life2Pat}$ (years)	t_{Life3} (years)	t_{Life} (years)	$ER_3(t)$
r	3.5%	40%	$2.88	11.33	9.76	8.28	11.47	29.37	0.420
r	**7.0%**	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
r	10.5%	-21%	$1.62	5.36	7.10	5.83	11.47	18.29	0.341
$\$_{oil}$	$27.5	-55%	$0.92	8.02	7.91	6.14	6.62	22.07	0.357
$\boldsymbol{\$_{oil}}$	$55.0	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
$\$_{oil}$	$82.5	56%	$3.21	6.82	8.02	6.86	14.31	21.70	0.375
$\$_{opex1}$	$4,528,512	1%	$2.09	7.50	8.04	6.77	11.47	22.31	0.375
$\boldsymbol{\$_{opex1}}$	$9,057,023	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
$\$_{opex1}$	$13,585,535	-1%	$2.04	6.79	8.04	6.77	11.47	21.61	0.368
$\$_{opex2Per}$	$10,566,527	2%	$2.10	6.79	8.72	6.77	11.47	22.29	0.372
$\boldsymbol{\$_{opex2Per}}$	$21,133,055	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
$\$_{opex2Per}$	$31,699,582	-2%	$2.02	7.48	7.44	6.77	11.47	21.69	0.371
$\$_{opex2Pat}$	$12,076,031	1%	$2.08	6.93	7.76	7.92	11.47	22.61	0.372
$\boldsymbol{\$_{opex2Pat}}$	$24,152,062	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
$\$_{opex2Pat}$	$36,228,093	-1%	$2.04	7.33	8.30	5.85	11.47	21.47	0.370
$\$_{opex3}$	$18,114,047	2%	$2.10	6.82	7.63	5.93	16.32	20.38	0.371
$\boldsymbol{\$_{opex3}}$	$36,228,093	**0%**	$2.06	**7.14**	**8.04**	**6.77**	**11.47**	**21.95**	**0.371**
$\$_{opex3}$	$54,342,140	-1%	$2.03	7.39	8.38	7.59	8.63	23.35	0.370

From **Table 22** we can infer the following results:

- Oil price has the largest impact of all the economic parameters over the NPV. As mentioned both the oil price and the OOIP multiply every inflow term of the NPV. The effect of this parameter on the production strategy is limited.
- The discount rate in real terms has a large impact on the NPV and the exploitation strategy. The smaller the discount rate the larger the NPV and the recovery efficiency. As pointed out in the base case analysis a different cost of capital can dramatically change the life of the project and the percentage of OOIP recovered. Note that the discount rate determines the time value of money, the smaller the discount rate the larger the life of the project.

- The NPV proves to be relatively insensitive to the operating cost. Remember that with a life cycle strategy we move to the next phase before reaching the economic limit at relatively large production rates within each phase. At these rates the outflow term of the cash flow is immaterial in comparison with the inflow. This is especially true for primary production were the operating costs are assumed to be \$3 per barrel as opposed to an oil price of \$55. The operating costs become more relevant as we move to pattern waterflooding (\$8 per barrel) and tertiary recovery (\$12 per barrel). However, the contribution of these phases to the NPV is relatively small. Operating costs have a large impact on the life of the project (not the NPV) for a myopic optimization where production continues until the economic limit is reached.
- Changes in the production strategy associated with variations in the operating cost are not significant with one exception, tertiary recovery. As we saw in section 4.3.3, when there are no more recovery mechanisms available, myopic and life cycle optimizations are equivalent. Therefore, it is expected that when tertiary costs are reduced by 50%, t_{Life3} lasts longer (from 11.47 to 16.32 years) and when they are increased the life of the project decreases (by 3 years).

4.5 Conclusion

This chapter evaluates, with a deterministic approach, the model introduced in Part A. We use the illustrative example presented in chapter 2 as a base case and maximize the NPV myopically, according to the base case production strategy, and over the life of the project. After analyzing the net present value, recovery efficiency and project life for the three cases we performed a sensibility analysis.

Our results suggest that time is dramatically reduced when the net present value is optimized for the life of the project. The base case clearly shows the greater economic efficiency of this life cycle approach. Total recovery efficiency decreases as a result of life cycle optimization.

The sensitivity analysis proves that the oil price and original oil in place are the most influential parameters. Discount rate in real terms and reservoir properties associated with primary recovery also play a key role. The NPV proves to be relatively insensible to tertiary recovery and operating costs when maximizing over the life of the project.

PART C: STOCHASTIC EVALUATION OF THE MODEL

The deterministic evaluation of the model in part B provides an optimization problem through the base case presented. Additionally, chapter 4 shows the effects that changes in the input parameters have over the net present value through a sensitivity analysis. Both the base case and the case study in the appendix are built on reservoir data known *a posteriori*.

In practice, field development decisions are made *a priori* when reservoir information is uncertain. To account for uncertainty part C presents a stochastic evaluation of the model. Chapter 5 introduces a decision analysis method while chapter 6 uses a Monte Carlo simulation.

Chapter 5: Decision Analysis for Two Phases

5.1 INTRODUCTION

This chapter develops a decision analysis framework to evaluate the expected value (EV) of the base case assuming the only alternatives available are primary and secondary production. To illustrate the decision problem the decision diagram and generic decision tree are presented before defining probability dependencies and the optimal strategy for the base case.

5.2 BACKGROUND ON DECISION ANALYSIS

Matheson and Howard (1968) defined decision analysis as

''a body of knowledge and professional practice for the logical illumination of decision problems." "The province of decision analysis is to be logical in complex, dynamic, and uncertain situations."

According to both authors

‘‘operations research was the first organized activity in the scientific analysis of decision-making. It originated in the application of scientific methods to the study of air defense during World War II.”

In the mid-1960s decision analysis

‘‘emerged as a discipline capable of treating the complexities of significant decision problem.” The essence of decision analysis is the decision to be made. “In describing decision analysis, the first step is to define a decision. In this report, a decision is considered an irrevocable allocation of resources, in the sense that it would take additional resources to change the action.”

The next step is to establish the nature of the decision problem and the variables affecting that decision.

5.3 CONSTRUCTING THE DECISION TREE FOR THE BASE CASE

5.3.1 The General Decision

Chapter 2 introduces the decision variables as the optimal times that should be devoted to each recovery phase. We simplified the decision to t_{Life1} or the time that should be allocated to primary production. Therefore, the decision to be made is whether to produce the reservoir with primary or secondary recovery methods. Let’s assume as well that this decision is made at year 0 or at the time of first oil and at year t. The decision at time t is essentially irrevocable. If we decide to stay in primary production we will not be able to change to secondary in consecutive years and vice versa.

Consequently, the two alternatives correspond to the production mechanisms available being primary or secondary recovery. The outcomes that this set of alternatives produce are the net present value associated to each production technique.

We selected as the system variables for the decision problem theoretical ultimate recovery efficiency (E_{R1}^{∞} *and* E_{R2}^{∞}) and the time constant for production (τ_1

and τ_2). Since both theoretical ultimate recovery and time constant for production are variables determined by the environment they are known as state variables. The nominal value and range of these parameters reflect the uncertainty assigned to the state variables.

> *"For convenience, we can often think of the nominal value of a state variable as its expected value in the mathematical sense and of the range as the 10th percentile and 90th percentile points of its probability distribution" Matheson and Howard, 1968.*

The nominal value, ranges and relationships among the system variables are discussed further in section 5.3.4 Defining Probability Distributions.

The final step is assigning a value to the outcome represented in this problem by the net present value. The relationship between the system variables and the net present value is defined in chapters 2 and 3. We use the analytical approach to calculate the outcomes or end points of the decision tree.

5.3.2 Decision Diagram

Decision diagrams are a graphical way to describe the dependencies among the system variables and decisions.

A decision diagram contains two types of nodes: *decision nodes* represented by boxes and *chance nodes* represented by circles or ellipses. Howard and Matheson distinguish two types of influences based on the arrows between node pairs. *Informational influences* or arrows leading into a decision node indicate "*which variables will be known at the time that the decision is made." Conditioning influences* or arrows leading to a chance node *"show the variables on which the probability assignment to the chance node variable will be conditioned."*

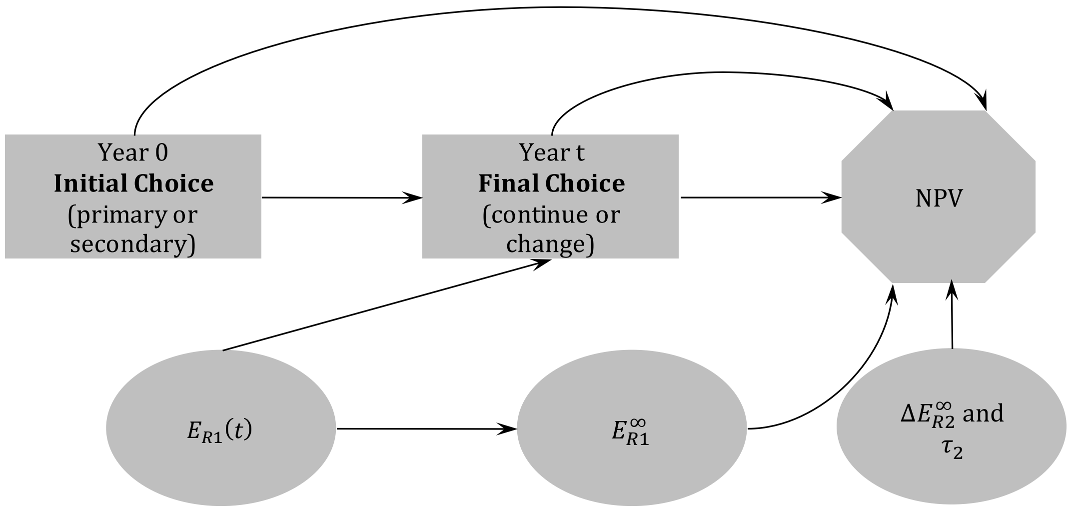

Figure 12: Decision diagram for two production phases

Figure 12 illustrates the decision diagram for two production phases. We can select between primary and secondary recovery methods at year 0 and t, observe the recovery efficiency at time t, choose a production alternative and obtained an NPV that depends on the ultimate recovery efficiency of the production method selected in years 0 and t.

The arrow between the decision nodes for year 0 and t shows that we know the decision in year 0 when the decision in year t is made. Similarly, the informational arrow between the recovery efficiency of primary production at time t and the final choice shows that we know the percentage of original oil in place recovered at time t when the decision in year t is made.

Conditioning influences are represented by the arrow between recovery efficiency of primary production at time t and theoretical ultimate recovery for primary production. This arrow shows that the theoretical ultimate recovery is conditioned upon the recovery efficiency at time t $\{E_{R1}^{\infty}|E_{R1}(t)\}$. Since we assume independence between primary and secondary recovery the theoretical ultimate recovery for secondary production does not depend on any other system variable. Finally the net present value depends on the decisions we made and the system variables.

5.3.3 Generic Decision Tree

The resulting generic decision tree from the decision diagram depicted in **Figure 12** appears in **Figure 13**. The two decision nodes indicate that we can select between primary and secondary recovery methods at year 0 and t. The chance nodes represent the system variables for primary and secondary production.

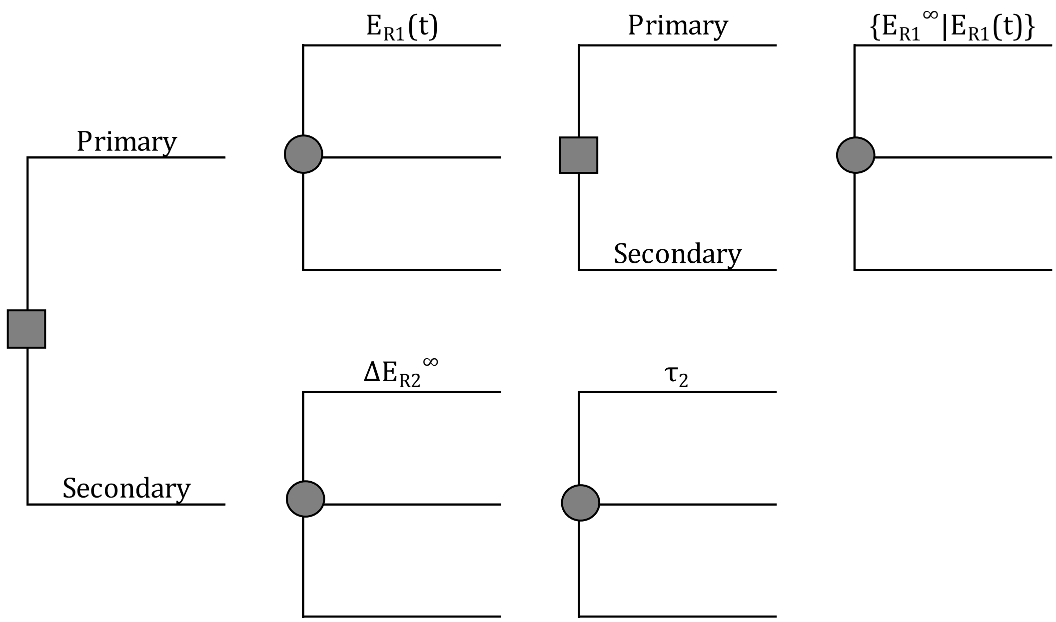

Figure 13: Generic decision tree for two production phases

The next step is to obtain the probability and a value assessment corresponding to the system or state variables.

5.3.4 Defining the Probability Distributions

We assign a triangular probability distribution for the theoretical ultimate recovery efficiency and the time constant for production for primary and secondary recovery[6]. **Table 23** shows the values assumed per parameter.

Table 23: State variables estimates

State Variable	Minimum	Most Likely	Maximum	Probability Distribution
E_{R1}^{∞} (fraction)	0.1	0.2	0.3	Triangular
τ_1 (years)	5	10	15	Triangular
ΔE_{R2}^{∞} (fraction)	0.05	0.15	0.25	Triangular
τ_2 (years)	2	5	7	Triangular

From Equation 3

$$E_{R1}(t) = E_{R1}^{\infty}\left(1 - e^{-t/\tau_1}\right)$$

Therefore, we can determine recovery efficiency for primary recovery at time t for any pair of E_{R1}^{∞} and τ_1. **Table 24** shows the detail for 10 of the 2,000 trials run for the simulation of recovery efficiency for primary recovery after 5 years of production.

[6] Considering that the estimates of the state variables are given in terms of the interval (a,b) and the most likely value (c) it seems logical to assume a triangular distribution.

Table 24: Summary of calculated recovery efficiencies for primary recovery

Trial	E_{R1}^{∞} (fraction)	τ_1 (years)	E_{R1}(t=5years) (fraction)
1	0.276	7.899	0.129
2	0.155	11.788	0.054
3	0.125	10.202	0.048
4	0.215	14.029	0.064
5	0.217	7.920	0.102
6	0.176	8.601	0.078
7	0.244	7.168	0.123
8	0.180	11.862	0.062
9	0.179	11.910	0.061
10	0.186	10.235	0.072

Randomly generated values for triangular distribution

Calculated from Equation 3

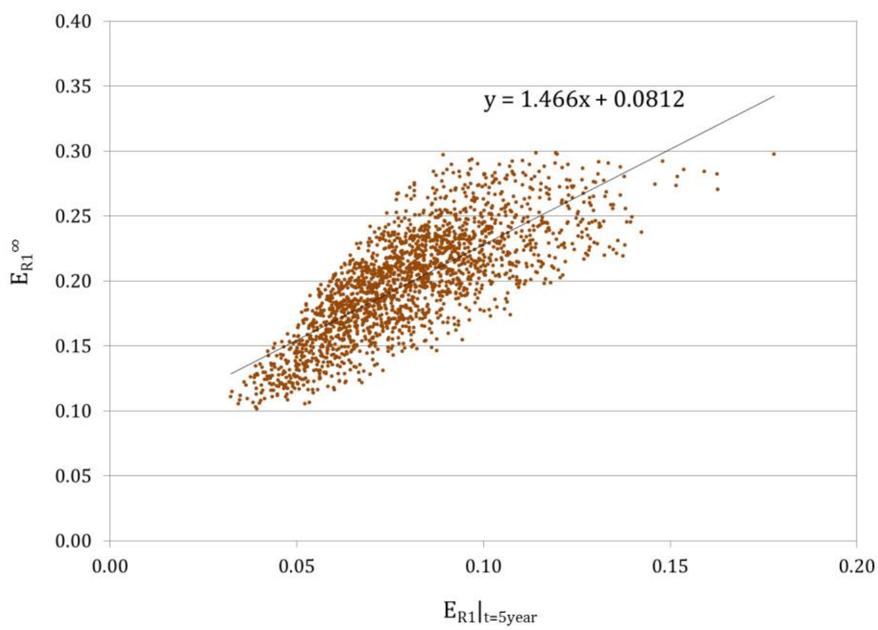

Figure 14: Conditional probability distribution of E_{R1}^{∞} given $E_{R1}(t = 5years)$

Figure 14 displays the results for 2,000 trials. The conditional probability of E_{R1}^{∞} given $E_{R1}(5)$ or $\{E_{R1}^{\infty}|E_{R1}(5)\}$ is specified by the regression line where

$$E_{R1}^{\infty} = 1.466 \cdot E_{R1}(5) + 0.0812$$

Equation 51

As stated previously we assume independence between theoretical ultimate recovery for primary and for secondary production. All we know about secondary production when we make our decision are the estimates displayed in **Table 23**.

5.3.5 A Three-Point Discretization Model

To include the state variables in the decision tree we approximate continuous uncertainty using a three-point discretization model. According to Bickel et al. (2010) the application of this method to the triangular distribution produces an approximation of 0.273, 0.454 and 0.273 for percentile 90, 50, and 10, respectively (see **Figure 15**).

P90	0.273
P50	0.454
P10	0.273

Figure 15: P10-P50-P90 approximations for triangular distribution

5.3.5.1A Three-Point Discretization Model for Primary Recovery

We start by determining the 90th, 50th and 10th percentile for primary recovery efficiency at year 5. For the 50th percentile we use the average value of $E_{R1}(5)$ obtained from the 2,000 iterations of E_{R1}^{∞} and τ_1 (**Table 24**). For the 10th and 90th percentile we assume

$$E_{R1}(5) = \overline{E_{R1}(5)} \pm 1.28\text{[7]} \cdot \sigma_{E_{R1}(5)}$$

Equation 52

where

$\overline{E_{R1}(5)}$ = the average of 2,000 trials for the recovery efficiency of primary production after 5 years.

$\sigma_{E_{R1}(5)}$ = the standard deviation of 2,000 trials for the recovery efficiency of primary production after 5 years.

Knowing that $\overline{E_{R1}(5)} = 0.0805$ (or 8.05% of original oil in place recovered) and $\sigma_{E_{R1}(5)} = 0.0215$ we obtain the values in **Figure 16**.

	$E_{R1}(5) = 0.108$
P90	0.273
	$E_{R1}(5) = 0.080$
P50	
	0.454
P10	$E_{R1}(5) = 0.053$
	0.273

Figure 16: Three point discretization for primary recovery at year 5

5.3.5.2A Three-Point Discretization Model for Theoretical Ultimate Recovery for Primary Production

The conditional probability of E_{R1}^{∞} given $E_{R1}(5)$ is specified by the regression line in Equation 51 where

$$E_{R1}^{\infty*} = 1.466 \cdot E_{R1}(5) + 0.0812$$

for the 50th percentile. Equation 53 shows the 10th and 90th percentile

$$E_{R1}^{\infty} = E_{R1}^{\infty*} \pm 1.28 \cdot \sigma_{E_{R1}^{\infty}}$$

---

[7] Recall that an 80% confidence interval is given by (sample estimate±1.28 standard errors).

Equation 53

where

$E_{R1}^{\infty *}$ = the conditional probability of E_{R1}^{∞} given $E_{R1}(5)$.

$\sigma_{E_{R1}^{\infty *}}$ = the standard deviation obtained from the regression analysis of **Figure 14**.

For example for $E_{R1}(5) = 8.05\%$

$$E_{R1}^{\infty *} = 1.466 \cdot 0.0805 + 0.0812 = 0.1992$$

with a $\sigma_{E_{R1}^{\infty *}} = 0.0264$ we determine the values for P90 and P10 (**Figure 17**).

	$E_{R1}^{\infty} = 0.233$
P90	0.273
	$E_{R1}^{\infty} = 0.199$
P50	
	0.454
P10	$E_{R1}^{\infty} = 0.165$
	0.273

Figure 17: Three point discretization for the theoretical ultimate recovery (primary production)

5.3.5.3 A Three-Point Discretization Model for Secondary Recovery

The 50th percentile for theoretical ultimate recovery efficiency for secondary recovery is the most likely value (refer to **Table 23**). Percentile 10th and 90th are equal to

$$\Delta E_{R2}^{\infty} = \overline{\Delta E_{R2}^{\infty}} \pm 1.28 \cdot \sigma_{E_{R2}^{\infty}}$$

Equation 54

where

$\overline{\Delta E_{R2}^{\infty}}$ = most likely value for the incremental theoretical ultimate recovery efficiency for secondary recovery.

$\sigma_{E_{R2}^{\infty}}$ = the standard deviation of the theoretical ultimate recovery efficiency for secondary production.

We proceed in a similar fashion with the time constant for production. Figures 18 and 19 show the results for ΔE_{R2}^{∞} and τ_2, respectively. The standard deviation for ΔE_{R2}^{∞} is 0.1 and for τ_2 is 2 years.

We can build now the decision tree for primary and secondary production.

P90	$\Delta E_{R2}^{\infty} = 0.278$	0.273
P50	$\Delta E_{R2}^{\infty} = 0.150$	0.454
P10	$\Delta E_{R2}^{\infty} = 0.022$	0.273

Figure 18: Three point discretization for theoretical ultimate recovery (secondary production)

P90	$\tau_2 = 7.56$	0.273
P50	$\tau_2 = 5.00$	0.454
P10	$\tau_2 = 2.44$	0.273

Figure 19: Three point discretization for the time constant for production (secondary production)

5.3.6 Decision Tree for Two Phases

Figure 20 shows the partial decision tree for the two phases.

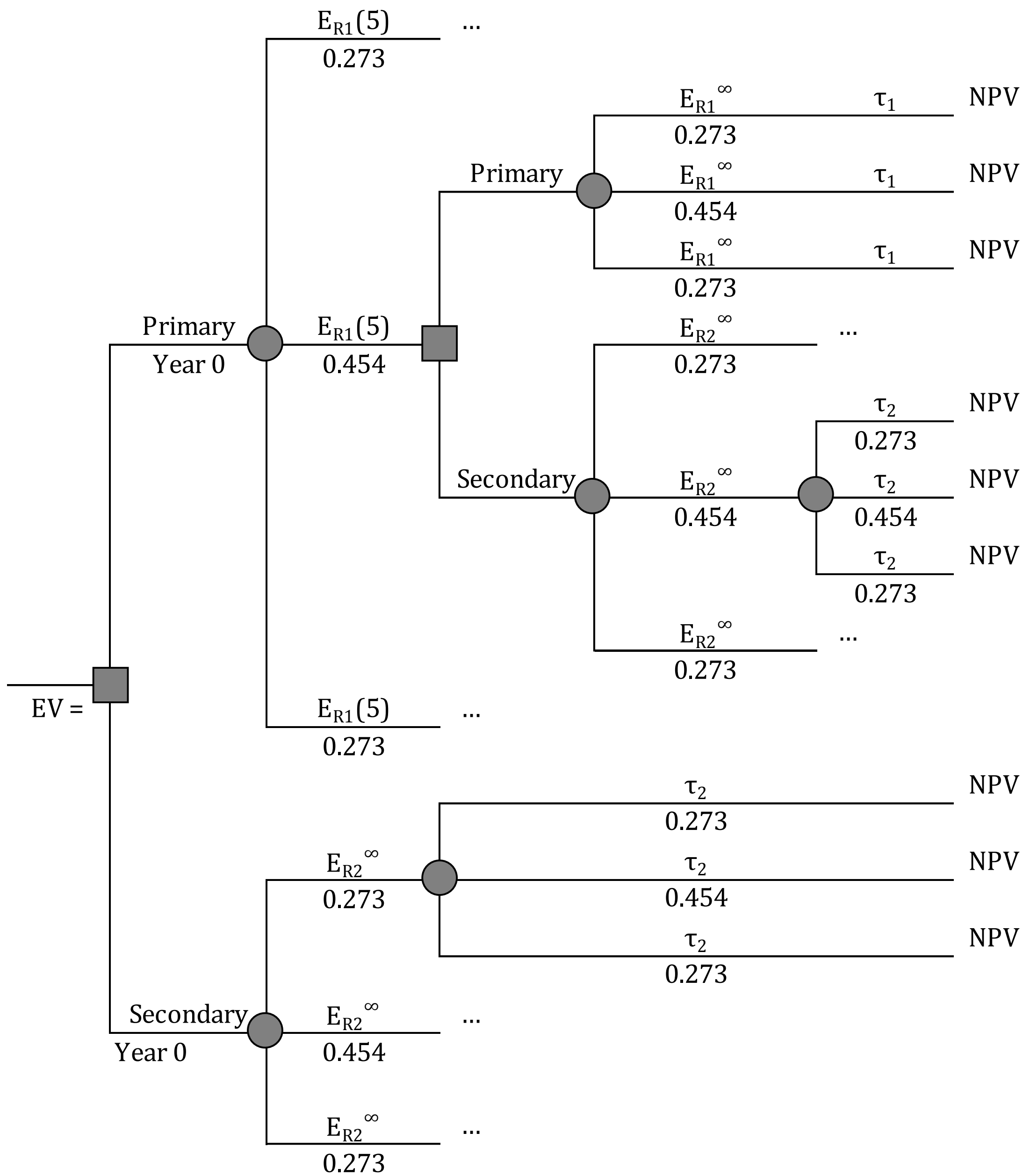

Figure 20: Partial decision tree for the two phase decision problem

The structure of the decision problem is straightforward. At year 0 we must decide to start production with primary or secondary recovery. If we select primary recovery at year 0 we can choose at year 5 whether to remain in primary for the life of the project or change to secondary based on the information available at the time (recovery efficiency at year 5). On the other hand, if we decide to start with secondary production at year 0 we remain in that phase for the entire life of the project.

Although the tree is conceptually simple it becomes complex even with a set number of years. In this example each year has 99 possible scenarios. We assume the theoretical ultimate recovery is reached at year 50, making the total number of end points equal to 99x50 or 4,950.

The following two sections reproduce sample calculations for primary and secondary recovery.

5.3.6.1Partial Decision Tree for Primary Production

Figure 21 shows the detail for the situation where the decision at year five is to continue primary production for the life of the project.

From Equation 3

$$E_{R1}(t) = E_{R1}^{\infty}\left(1 - e^{-t/\tau_1}\right)$$

Therefore, the time constant for production (τ_1) can be obtained from the theoretical ultimate recovery and the recovery efficiency for primary production at time t according to the following relationship

$$\tau_1 = -t/ln\left[1 - \left(\frac{E_{R1}}{E_{R1}^{\infty}}\right)\right]$$

Equation 55

For example for $E_{R1}(5) = 8.05\%$ and $E_{R1}^{\infty} = 19.92\%$

$$\tau_1 = -5/ln\left[1 - \left(\frac{0.0805}{0.1992}\right)\right] = 9.66$$

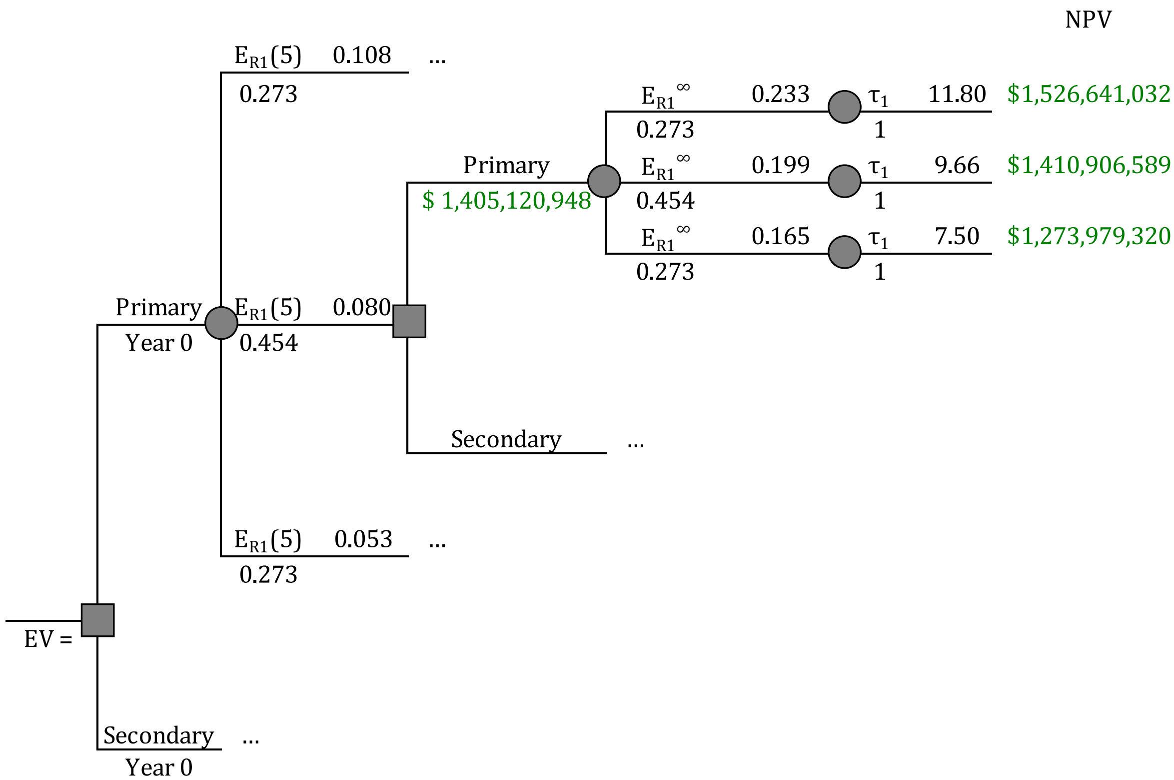

Figure 21: Partial decision tree for the alternative "primary production for the life of the project"

The end points correspond to the NPV associated with the values of that event. Using Equation 42 and the data from the illustrative example in chapter 2 we have

$$NPV_1 = (2.4 \cdot 10^8)(55)(0.1992)\left(1 - e^{-\frac{1}{9.66}}\right)\left[\frac{1 - \left(\frac{e^{-\frac{1}{9.66}}}{1+0.07}\right)^{50}}{1 + 0.07 - e^{-\frac{1}{9.66}}}\right] - \frac{9{,}057{,}023}{0.07}\left[1 - \left(\frac{1}{1+0.07}\right)^{50}\right] = \$1{,}410{,}906{,}589 = \$1.4\ bn$$

Similarly, we find the NPV for P10 and P90. The expected value for the primary recovery alternative is equal to the probability of occurrence of a given event time the NPV or

$$E(Primary) = 0.273 \cdot \$1{,}526{,}641{,}032 + 0.454 \cdot \$1{,}410{,}906{,}589 + 0.273 \cdot \$1{,}273{,}979{,}320 = \$1{,}405{,}120{,}948$$

5.3.6.2 Partial Decision Tree for Secondary Production

Figure 22 shows the detail where the decision at year zero is to start with secondary production and continue with this strategy for the life of the project.

Likewise, the end points correspond to the NPV associated with the values of that event. We use Equation 42 as well since there is only one recovery mechanism. The expected value of this alternative is $1,1 bn.

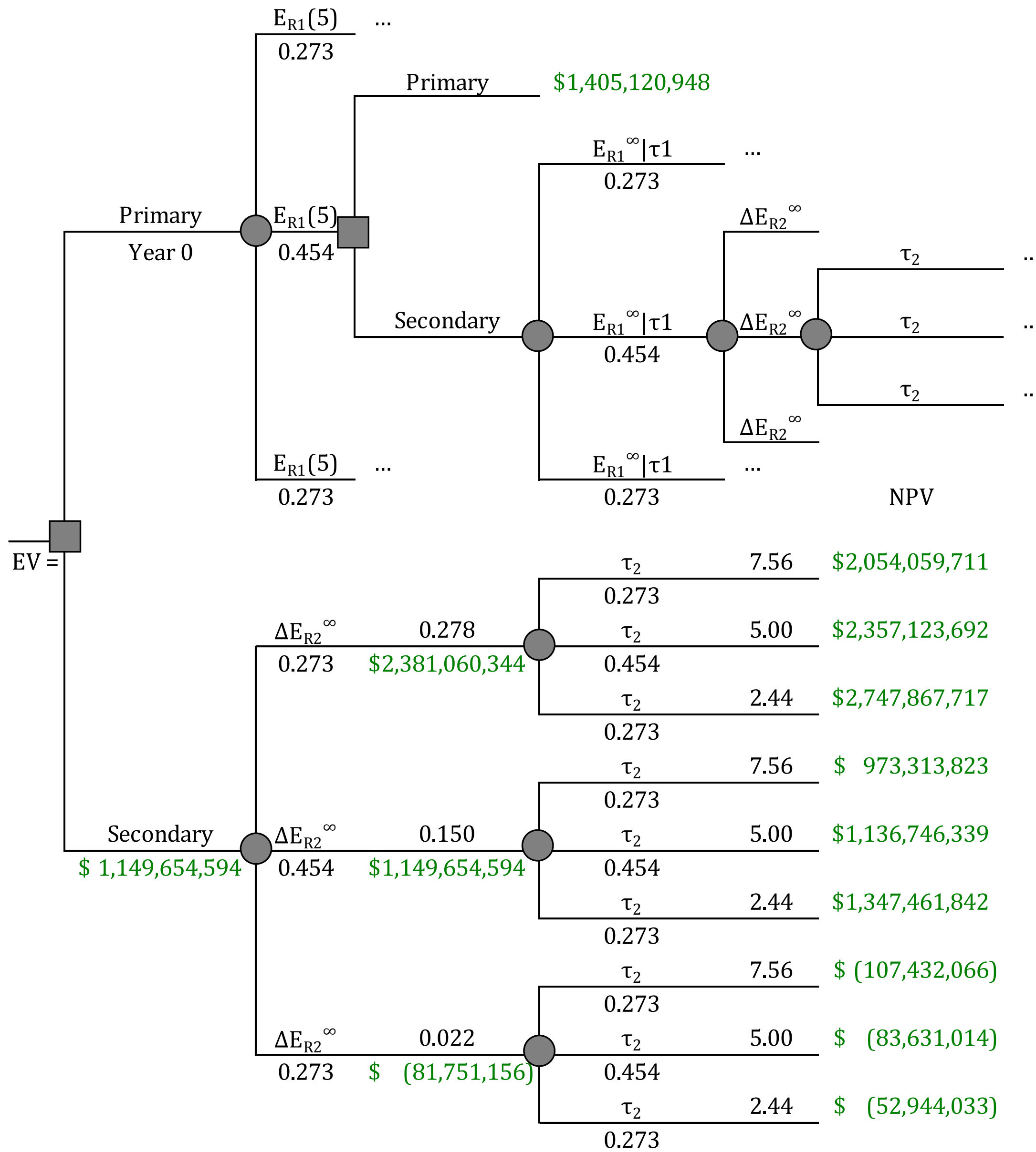

Figure 22: Partial decision tree for the alternative "secondary production for the life of the project"

5.3.6.3 Partial Decision Tree for Two Phases

Figure 23 shows the expected value of each alternative assuming we decide in year 5 whether to continue with primary production or move to secondary. We have already explained the alternatives of only primary or only secondary production for the life of the project. However, we could start with primary and after 5 years move to secondary production.

As before, the end points correspond to the NPV associated with the values of that event. We use Equation 42 for primary production for the first five years and Equation 47 for the contribution of secondary production during the next 45 years.

The probability of each event times the NPV gives an expected value for this alternative of $1,669,259,634. Since this number is larger than the expected value of continuing with primary production ($1,405,120,948) is it advisable to start secondary production after 5 years with primary recovery. Following the same reasoning the first decision node indicates that in year 0 we should start with primary production as the expected value ($1,669,477,089) is larger than using secondary for the life of the project ($1,149,654,594).

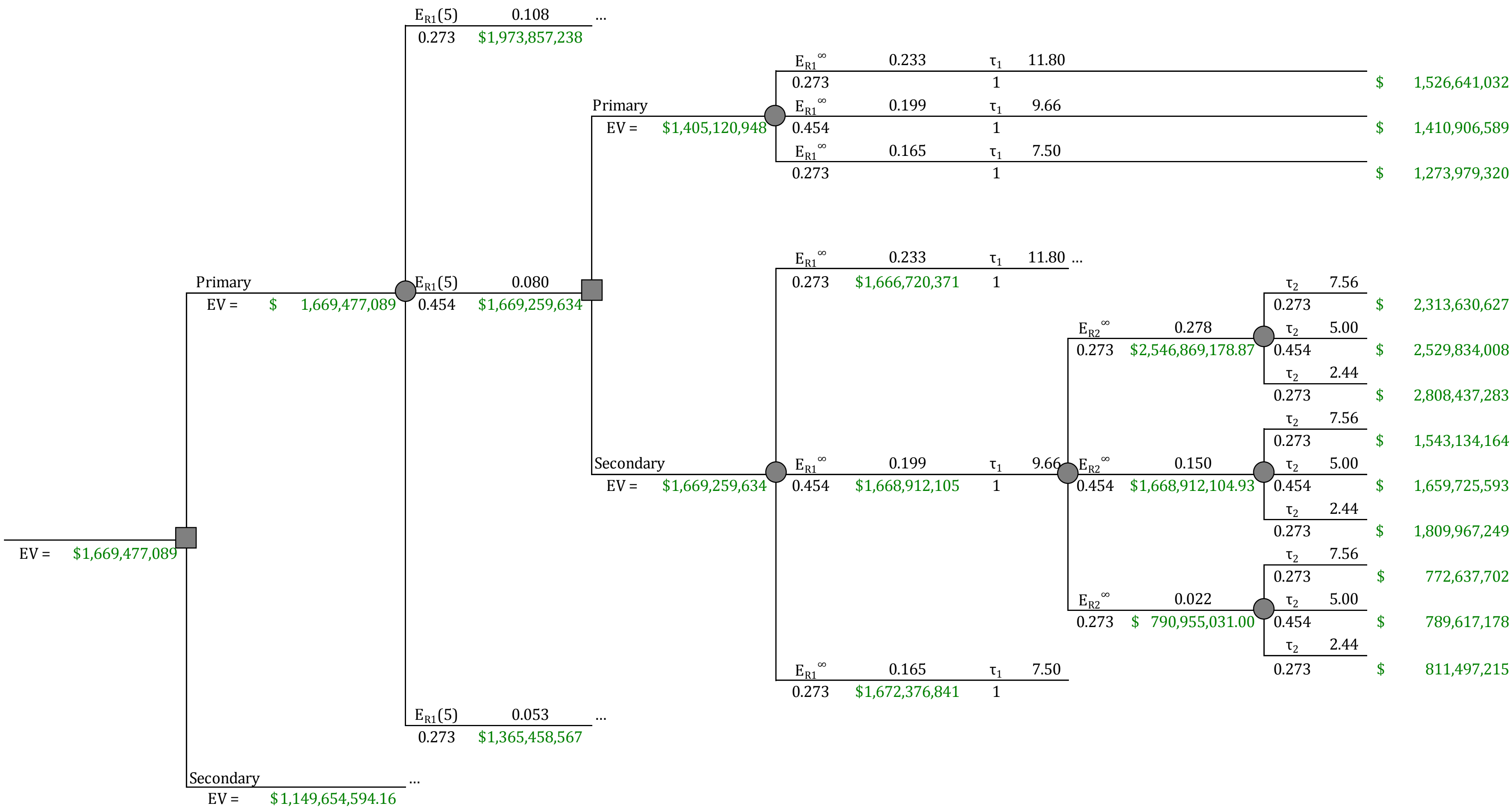

Figure 23: Partial decision tree for two phases

5.4 DETERMINING THE OPTIMAL STRATEGY

We assume the theoretical ultimate recovery is reached at year 50 ($E_{R1}^{\infty} = E_{R1}(50)$). Therefore, we reproduce the calculations and decision tree of t=5 for all the years.

	Final choice	($bn)
	Year 1	$ 1.473
	Year 2	$ 1.490
	Year 3	$ 1.560
	Year 4	$ 1.621
	Year 5	$ 1.669
	Year 6	$ 1.705
	Year 7	$ 1.737
	Year 8	$ 1.763
	Year 9	$ 1.779
	Year 10	**$ 1.791**
	Year 11	$ 1.782
Optimal switching time	...	...
	Year 40	$ 1.529
	Year 41	$ 1.523
	Year 42	$ 1.515
	Year 43	$ 1.565
	Year 44	$ 1.523
	Year 45	$ 1.555
	Year 46	$ 1.540
	Year 47	$ 1.551
	Year 48	$ 1.537
	Year 49	$ 1.525
	Year 50	$ 1.493

Figure 24: NPV expected value per switching time

Figure 24 shows the NPV expected values obtained from the different decision trees. The column final choice indicates the year in which we can decide whether to continue in primary production or switch to secondary production. **Figure 25** displays the same results for all the years. From year 1 to 10 the expected NPV increases as the alternative to switch to another production mechanism is delayed in time. The highest value is reached in year 10 with an expected NPV of $1.791 billion dollars. After 10 years we observed a slow decline that continues until year 32 and from this period to 50 years the NPV stabilizes around $1.5bn.

These results are consistent with the ones obtained in chapter 4 for the base case using a deterministic approach. The optimal time in that case was 11 years. The difference between the two solutions would be less than a year if the decision tree would contemplate results per month as the maximum is between year 10 and 11.

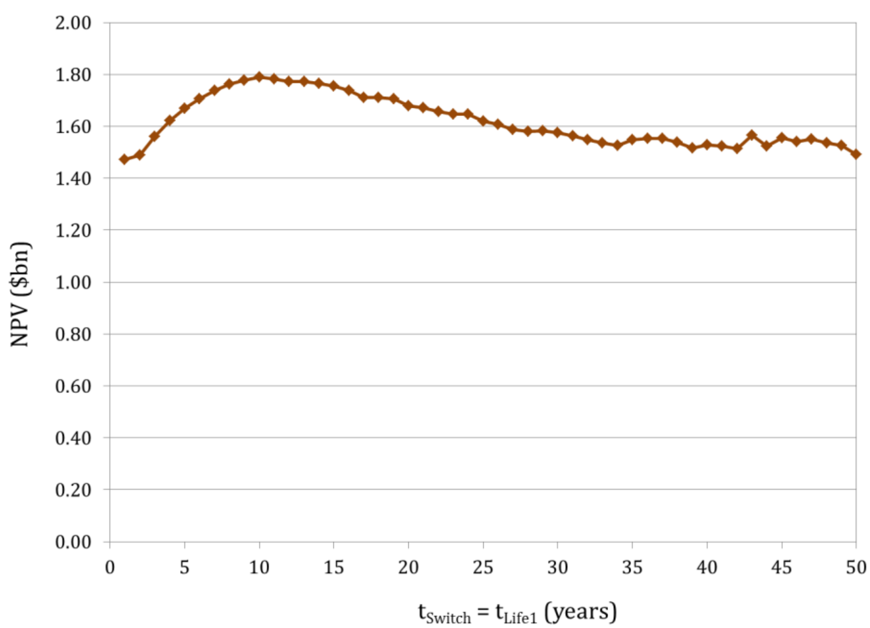

Figure 25: NPV expected value as a function of switching time

5.5 CONCLUSION

This chapter introduces uncertainty to the optimization problem through a decision analysis framework. To achieve the initial goal of making a reasoned decision that maximizes the net present value subject to the time assigned to each recovery phase, we build a decision tree for primary and secondary production. First we determine the state variables and its probability of occurrence. By repeating the analysis for different years we obtain the optimal time in which secondary production should start. Although the decision analysis is applied to two production phases it can be extended to tertiary recovery.

The next chapter presents uncertainty propagation through a Monte Carlo simulation.

Chapter 6: Monte Carlo Simulation

6.1 Introduction

According to Saputelli et al. *"uncertainty propagation implies the use of a stochastic simulation method, such as Monte Carlo. Stochastic simulation is a technique to propagate uncertainty on a mathematical model by repetitive and random perturbation on each of its uncertain inputs. In this technique, each variable's probability distribution function is sampled at random intervals."*

By using a stochastic method a new reservoir is generated for each iteration. For this simulation we have performed 1,000 iterations per case. The next sections contain the simulation procedure and the results assuming independent and dependent recovery efficiencies.

6.2 General Description of the Optimization Procedure

The first step in a stochastic simulation is to define the uncertain parameters and the range of values. The main uncertainties of a field are related to the reservoir properties. Recovery efficiency is a function of the theoretical ultimate recovery and the time constant for production. Both of these variables depend strongly on the reservoir properties and how the reservoir is produced. For the Monte Carlo method, the theoretical ultimate recovery and the time constant for production were modeled with a log-normal distribution function. The average numbers for ultimate primary, secondary, and tertiary production were set according to Walsh and Lake (2003) expected values for the average commercial oil reservoir.

The next step was to generate inputs randomly from the domain. First, we considered independence between recovery efficiencies. A later section dependence between primary, secondary, and tertiary production was assumed.

Once the values were generated we performed a deterministic optimization based on each iteration. Following the same procedure as in chapters 3 and 4 the optimization was done myopically, or maximizing the net present value per recovery phase; and with a life cycle approach, or maximizing the net present value over the life of the project. The optimizations were solved using Solver® in Excel.

The last step was to add the solutions of the individual computations into the final result. Since the objective function (net present value) is nonlinear the results were first analyzed based on the average values of the uncertainties for primary, secondary, and tertiary production; and later on the average values of the NPV and decision variables (time allocated to each recovery phase).

6.3 BASE CASE ASSUMING INDEPENDENCE BETWEEN RECOVERY EFFICIENCIES

6.3.1 Selecting Uncertain Parameters and Their Probability Distribution

Table 25 shows the probability distribution function, the parameters and the ranges assumed for the stochastic model.

The variables consist of the following:

- E_{R1}^{∞} theoretical ultimate recovery efficiency for primary recovery
- τ_1 time constant for production for primary recovery
- ΔE_{R2}^{∞} incremental theoretical ultimate recovery efficiency for secondary recovery
- τ_2 time constant for production for secondary recovery
- ΔE_{R3}^{∞} incremental theoretical ultimate recovery efficiency for tertiary recovery

τ_3 time constant for production for tertiary recovery

Table 25: Probability distribution for reservoir variables assuming independence between recovery efficiencies

Variable	Average	Standard Deviation
E_{R1}^{∞} (fraction)	0.152	0.069
τ_1 (years)	7	3
ΔE_{R2}^{∞} (fraction)	0.199	0.046
τ_2 (years)	10	5
ΔE_{R3}^{∞} (fraction)	0.111	0.066
τ_3 (years)	7	3

6.3.2 Results

The next section present the results for primary, secondary, and tertiary recovery obtained from the expected value of the uncertainties and the recovery efficiencies. As mentioned earlier because to the nonlinearity of the objective function the average NPV is larger than the NPV obtained from the mean value of the uncertainty variables and the recovery efficiencies at the optimal times. Therefore, we analyze first the results for the expected values of the uncertainties for primary, secondary, and tertiary production; and later the average values of the NPV and decision variables.

To graph the results we used the numerical approach developed in chapter 2 and assumed the same values for oil price, original oil in place and discount rate in real terms (**Table 26**).

Table 26: Original oil in place, discount rate and assumed oil price

Variable	Value	Units
N	240,000,000	bbls
r	7%	yearly rate
$\$_{oil}$	55	USD

Similarly, we determine the yearly operating cost per recovery phase base on the average production from year 1 through 10 and the cost per barrel include in **Table 27**.

Table 27: Cost estimates

Recovery Phase$_i$	Opex ($/bbl)	$\$_{Opexi}$
1. Primary	3	$8,211,769
2. Secondary	8	$24,273,429
3. Tertiary	12	$24,087,855

6.3.2.1 Myopic and Life Cycle Optimization for Primary Recovery

For the first case we assume that the only recovery mechanism available at the time the decision is made is primary production. Hence, the myopic and the life cycle optimization are equivalent methods. Maximizing the NPV per recovery phase is the same as maximizing the NPV over the life of the project when only one phase is available.[8]

[8] We use a similar analysis if we optimize over a field in which only tertiary recovery is an option. In that case myopic and life cycle optimization are equivalent and production would continue until the economic limit is reached.

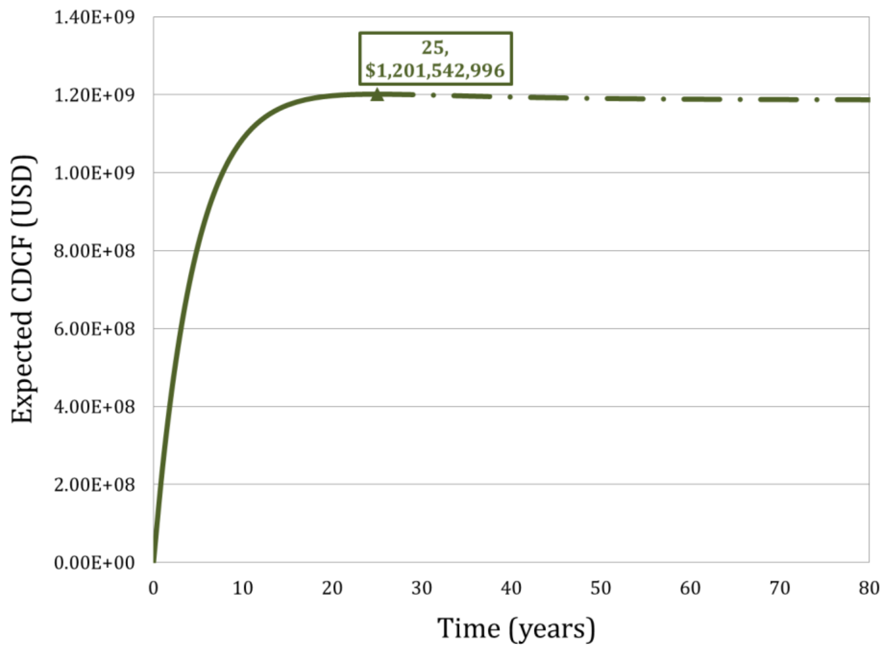

Figure 26: Expected cumulative discounted cash flow as a function of time (primary production)

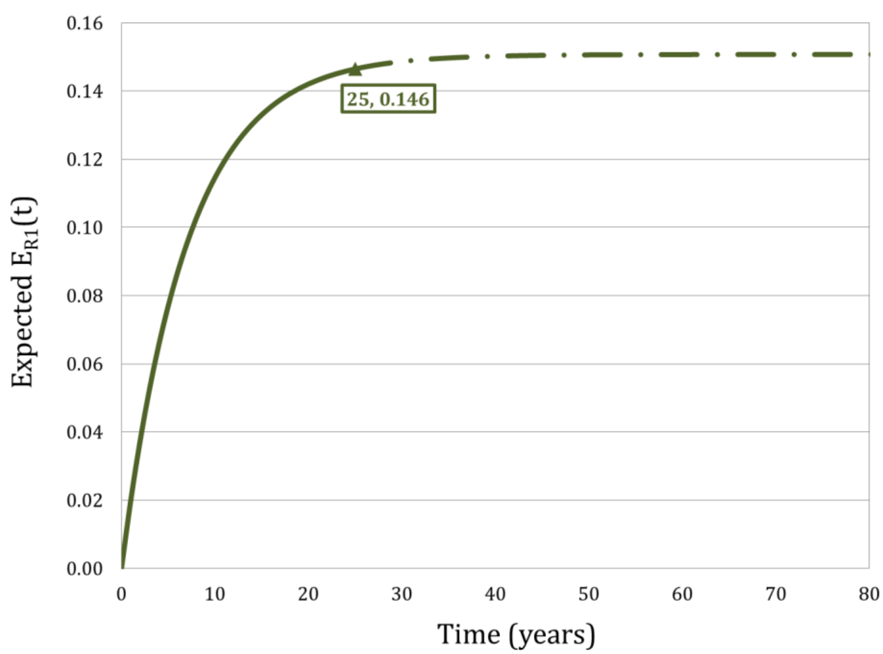

Figure 27: Expected recovery efficiency as a function of time (primary production)

Figure 26 presents the cumulative discounted cash flow (CDCF) as a function of time. The CDCF in dollars is graphed on the y axis and the time in years is on the x axis. If only one recovery technique is available production would last 25 years or until the economic limit is reached. The optimized net present value of this scenario would be $1.2 bn.

Figure 27 shows the recovery efficiency for primary production as a function of time. Recovery efficiency is plotted on the y axis and time in years is on the x axis. After 25 years of production only 14.65% of the original oil in place would be recovered.

6.3.2.2 Myopic and Life Cycle Optimization for Primary and Secondary Recovery

Table 28 summarizes the results for a myopic and a life cycle approach when primary and secondary production are available. We see that the NPV for the life cycle optimization is over 20% larger than the NPV for the myopic optimization. We observe as well a 17 year reduction in the life of the project and 4.5% decrease in the percentage of oil recovered when maximizing the NPV for the life of the project (0.327-0.282).

Table 28: Myopic versus life cycle optimization for two phases

Expected values	Myopic Optimization	Life Cycle Optimization
NPV ($bn)	$1.43	$1.72
t_{Life} (years)	49	32
Cummulative Recovery Efficiency	0.326	0.282

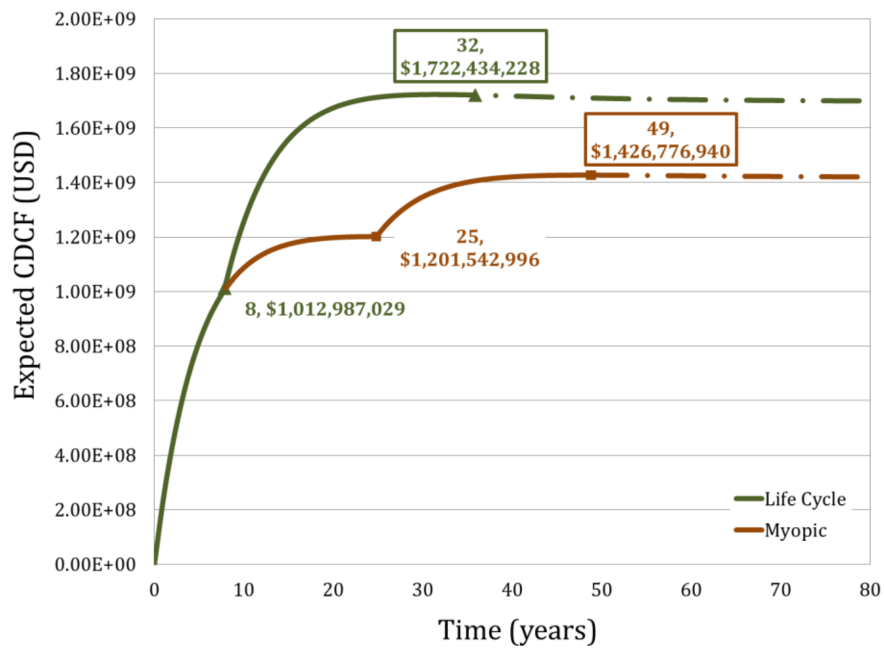

Figure 28: Expected cumulative discounted cash flow as a function of time (primary and secondary production)

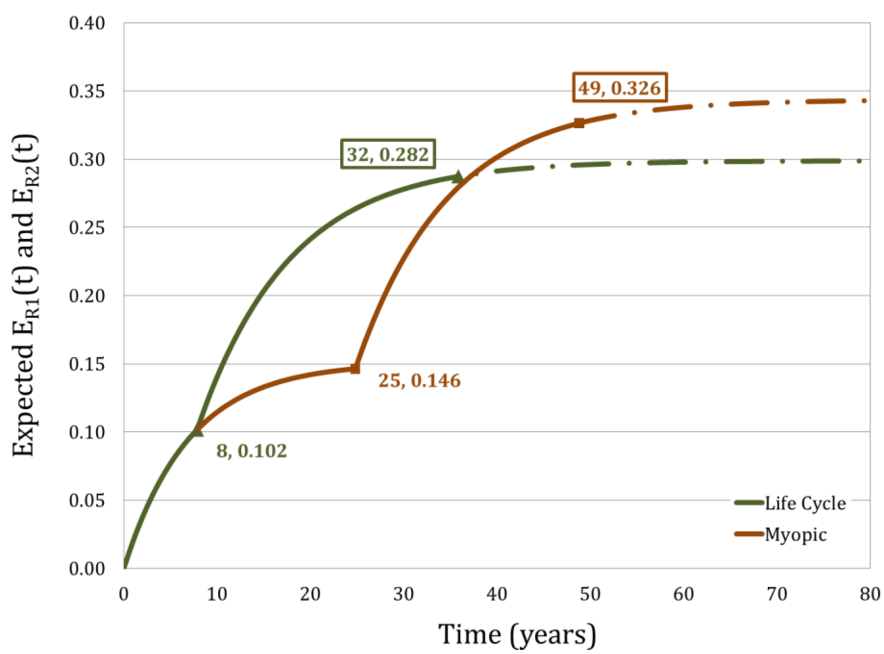

Figure 29: Expected recovery efficiency as a function of time (primary and secondary production)

Figure 28 shows the net present value for each phase. When using a life cycle optimization, primary production lasts for 8 years and accounts for 59% of the NPV. Secondary recovery continues for 24 years and adds 41% to the NPV of the project.

The contribution of primary production to the NPV is larger for the myopic approach with 84% in 25 years. Secondary production only contributes 16% to the NPV when maximizing per recovery phase.

Secondary production lasts 24 years in both optimization methods. Once secondary recovery starts production continues until the economic limit is reached for life cycle and myopic optimization.

Recovery efficiency as a function of time is plotted in **Figure 29**. We see that the incremental recovery of secondary production is 18.01% (28.17%-10.16% or 32.65%-14.64%) for both optimizations. Therefore, the 4.48% reduction for the life cycle approach is entirely because the smaller recovery obtained in primary production. After 8 years, recovery efficiency and NPV become competing criteria. Additional production increases recovery efficiency to the detriment of net present value and vice versa. As a result, the time assigned to the first recovery mechanism becomes critical when maximizing returns.

6.3.2.3 Myopic and Life Cycle Optimization for Primary, Secondary and Tertiary Recovery

Table 29 presents a summary of the results for three phases. The net present value is consistently larger when maximized over the life of the project. As we add a third phase the difference between the NPVs from the two optimizations increases.

Moreover, the life cycle optimization leads to a shorter project life than the myopic optimization and the total recovery efficiency decreases as a result of this approach.

Table 29: Myopic versus life cycle optimization for three phases

Expected values	Myopic Optimization	Life Cycle Optimization
NPV ($bn)	**$1.45**	**$1.87**
t_{Life} (years)	**63**	**34**
Cummulative Recovery Efficiency	**0.426**	**0.335**

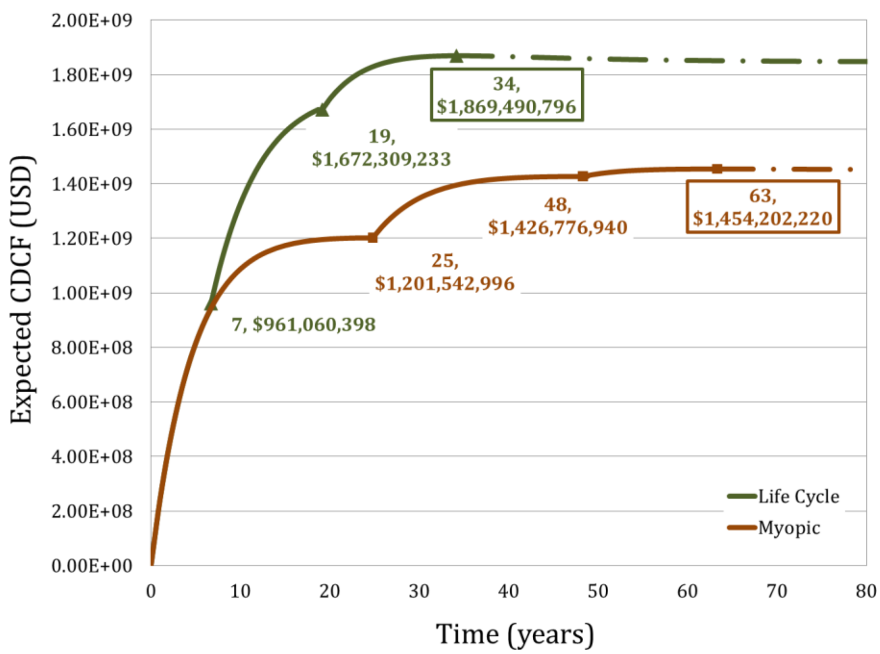

Figure 30: Expected cumulative discounted cash flow as a function of time (primary, secondary and tertiary production)

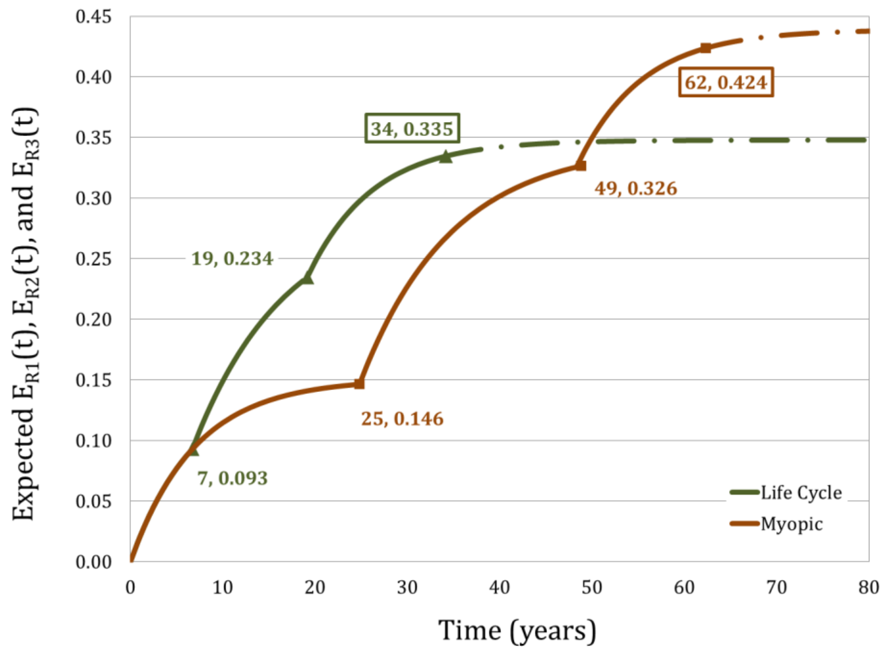

Figure 31: Expected recovery efficiency as a function of time (primary, secondary, and tertiary production)

Figure 30 displays the CDCF as a function of time. The life cycle optimization assigns 7, 12 and 15 years to primary, secondary, and tertiary recovery, respectively. Primary production contributes 51% to the NPV of the project. Secondary production accounts for 38% and tertiary for 11% bn.

When optimizing myopically, primary, secondary and tertiary production continue for 25, 23 and 15 years. Primary production contributes 83% to the NPV of the project. Secondary production accounts for 15% and tertiary for 2%.

We observe from the incremental NPV results that the time assigned to the first recovery method is key even when a third phase is available. Chapter 2 mentioned that the time value of money is taken into account by discounting the cash flow. Money held now is more valuable than money received at some future date. As the decision of implementing a new recovery method is further in time, its discounted value decreases. In this case, after 15 years the contribution of tertiary recovery is only $0.20 bn for the life cycle optimization and $0.03 for the myopic optimization.

Figure 31 shows the recovery efficiency for both optimizations. Total recovery efficiency decreases by 8.9% as a result of the life cycle optimization.

6.3.3 Net Present Value and Optimal Times Results

6.3.3.1 Comparing Myopic and Life Cycle Results

We see the same trends for the average NPV and t_{Life} obtained from the different iterations.

Figure 32 summarizes the results of the NPV when one, two or three phases are available. If primary recovery is the only production mechanism both optimizations lead to the same results. Adding a second recovery phase provides a larger contribution to the average NPV than including a third phase.

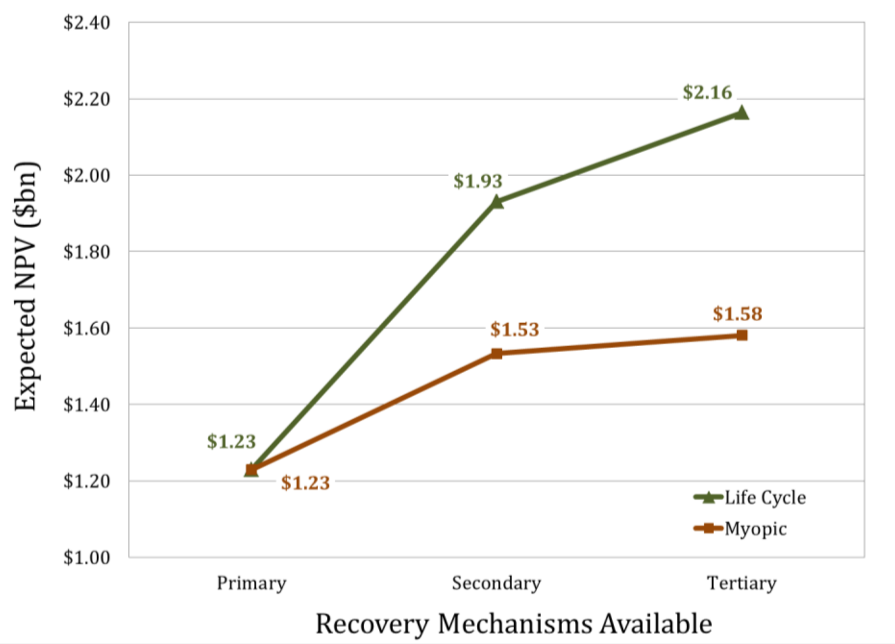

Figure 32: Expected NPV for life cycle and myopic optimization

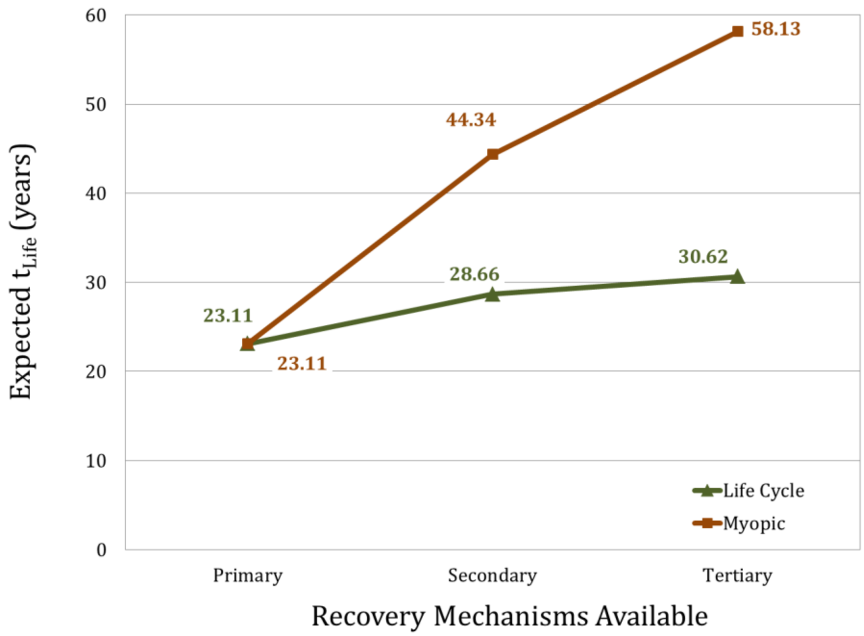

Figure 33: Expected life of the project for life cycle and myopic optimization

Figure 33 compares the life of the project for both myopic and life cycle optimization when one, two or three phases are available. The difference between the two methods increases from two to three recovery phases. The results of the life cycle optimization show minor changes when adding tertiary recovery.

Figure 34 illustrates the time assigned to each phase for the life cycle optimization. As we discussed in previous charts, production continues until we reach the economic limit for the last recovery mechanism available in the life cycle optimization. Consequently if the only option is primary recovery, a life cycle and a myopic optimization lead to the same results. When secondary production is added the time devoted to primary recovery, t_{Life1} decreases abruptly from 23.11 to 7.42 years. However, if a third phase is considered the change in t_{Life1} is less than 2 years. Assuming a new technology was discovered after we start production we would not expect significant changes on the time devoted to primary and secondary recovery.

It would be the last mechanism available, in this case tertiary recovery, the one that could potentially experience the largest decrease from the initial 13.79 years.

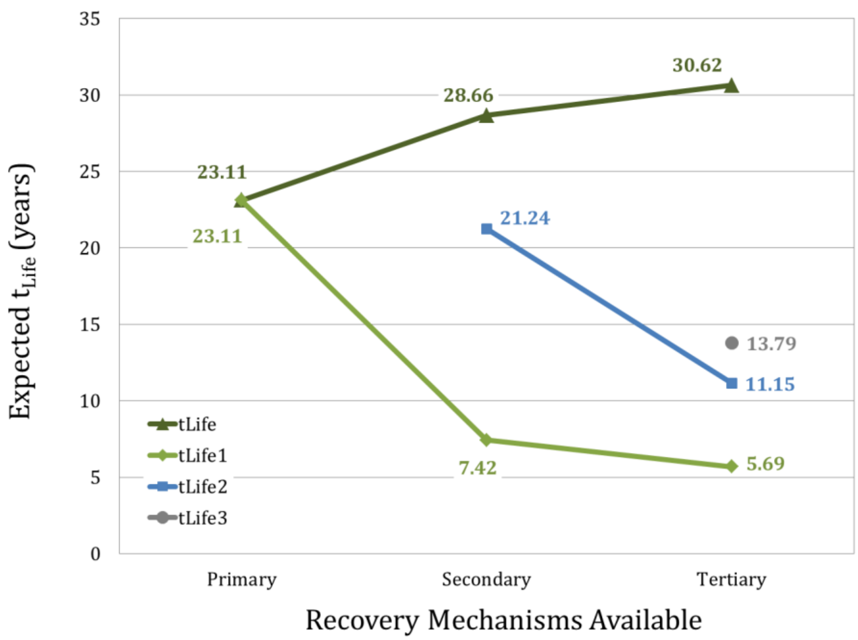

Figure 34: Expected life of the project and t_{Lifei} per recovery phase for life cycle optimization

Figure 35 shows the average life of the project and the time allocated to each recovery phase for myopic optimization. When maximizing the NPV per recovery phase the time assigned per phase is independent of the recovery mechanisms available. Hence, we see that t_{Life1} and t_{Life2} remain constant.

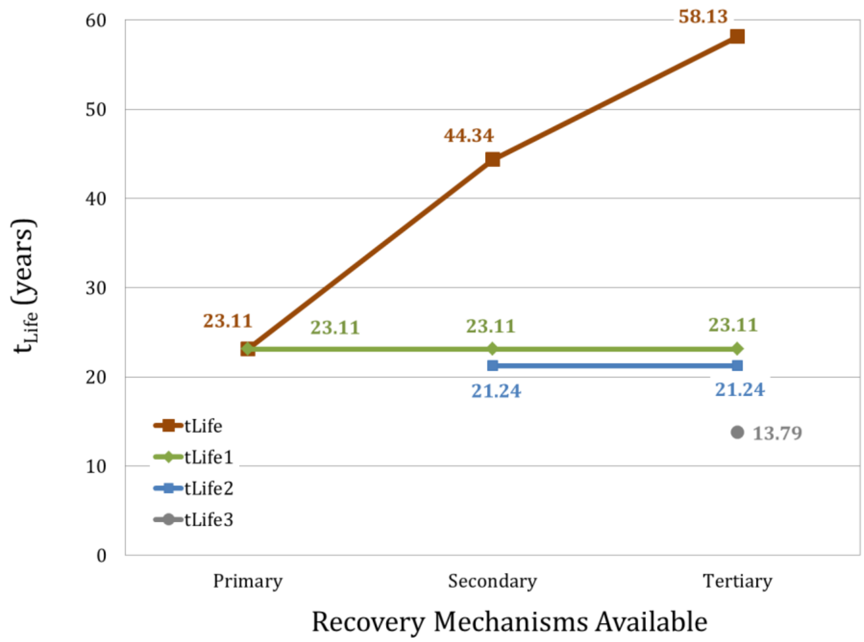

Figure 35: Expected life of the project and t_{Lifei} per recovery phase for myopic optimization

6.3.3.2 Analyzing Correlation Coefficients of Optimized Key Parameters

The linear correlation coefficients between the uncertainties, the decision variables and the objective function are presented in **Table 30**. We assume dependence on any pair of variables with a correlation coefficient greater or equal to |0.4| (in bold).

First we evaluate the variables affecting the objective function or NPV. Both theoretical ultimate recovery for primary and for secondary production have a large impact on the NPV. The influence of the theoretical ultimate recovery for tertiary production is limited.

Negative correlation coefficients are related to primary and secondary time constants for production. These results should not come as a surprise. We have seen that the contribution of tertiary recovery to the project NPV is minimal. As the decision of implementing a new recovery method is further in time its discounted value decreases. Same reasoning applies for the time constant for production for primary and secondary recovery (τ_1 and τ_2). Given the same values for primary and secondary theoretical ultimate recovery the reservoir with smaller τ_1 and τ_2 will have a larger NPV. Recall that time constant indicates how long it takes to reach certain recovery efficiency. The longer the time required, the smaller the discounted value.

Table 30: Linear correlation coefficients assuming independence for life cycle optimization

	E_{R1}^{∞}	τ_1	ΔE_{R2}^{∞}	τ_2	ΔE_{R3}^{∞}	τ_3	t_{Life1} (years)	t_{Life2} (years)	t_{Life3} (years)	t_{Life} (years)	NPV_{LC} ($)
E_{R1}^{∞} (fraction)	1	0.00	0.03	0.00	0.03	-0.03	**0.68**	0.00	0.00	**0.41**	**0.63**
τ_1 (years)		1	-0.04	-0.02	-0.05	0.04	0.00	0.01	0.00	0.01	-0.22
ΔE_{R2}^{∞} (fraction)			1	-0.02	-0.02	-0.05	**-0.41**	**0.61**	-0.03	0.28	**0.61**
τ_2 (year)				1	0.00	-0.01	0.26	0.04	0.00	0.19	-0.22
ΔE_{R3}^{∞} (fraction)					1	0.05	-0.21	**-0.55**	**0.71**	-0.12	0.27
τ_3 (year)						1	0.03	0.17	**0.60**	**0.59**	-0.09
t_{Life1} (years)							1	-0.11	-0.13	**0.41**	0.03
t_{Life2} (years)								1	-0.32	**0.59**	0.15
t_{Life3} (years)									1	0.33	0.10
t_{Life} (years)										1	0.22
NPV ($)											1

Note: In bold are the correlation coefficients ≥|0.4|

For decision variables we observe the following dependencies:

- The time devoted to primary recovery is directly proportional to the ultimate recovery efficiency for primary and the time constant for production and inversely proportional to the ultimate recovery efficiency for secondary and tertiary. In other words the time allocated to primary production depends not only in the efficiency of this recovery technique but also on consecutive phases. In general we move to secondary production faster if we expect high returns from that recovery stage.

- We observe a similar behavior for the time assigned to secondary production. If we expect large efficiency from tertiary recovery the life of secondary will be shorter.
- The time devoted to tertiary recovery is also a function of the recovery efficiency for this phase. Additionally, the time constant for production is directly proportional to t_{Life3} as there are no production alternatives after tertiary.
- Last, the life of the project increases with ultimate recovery efficiency for primary and secondary production and decreases with large efficiencies for tertiary recovery. The time constant for production for tertiary recovery plays a key role. We expect longer project life when the efficiency of primary recovery and τ_3 are large.

6.4 VARIATION OF THE BASE CASE ASSUMING DEPENDENCE BETWEEN RECOVERY EFFICIENCIES

We completed the same stochastic simulation shown in section 6.3 assuming dependence between recovery efficiencies. In general we expect secondary recovery to perform better when primary performance is poor and vice versa. Likewise, tertiary production should be more efficient when the results for secondary are low.

6.4.1 Selecting Uncertain Parameters and Their Probability Distribution

Dependence between recovery efficiencies was introduced in the generation of random inputs according to the relationships indicated in the first column of **Table 31**.

Table 31: Probability distribution for reservoir variables assuming dependence

	Parameters	Average	Standard Deviation
(1)	$E_{R1}^{\infty} + \Delta E_{R2}^{\infty}$ (fraction)	0.310	0.050
(2)	$\Delta E_{R2}^{\infty} + \Delta E_{R3}^{\infty}$ (fraction)	0.350	0.050
(3)	ΔE_{R2}^{∞} (fraction)	0.200	0.050
(1)-(3)	E_{R1}^{∞} (fraction)	0.152	0.070
(2)-(3)	ΔE_{R3}^{∞} (fraction)	0.111	0.066
	τ_1 (years)	7	3
	τ_2 (years)	10	5
	τ_3 (years)	7	3

We first generated values for the ultimate recovery efficiency of primary and secondary production (first row) and for the ultimate recovery efficiency of secondary and tertiary production (second row). We then obtain random values for the incremental ultimate recovery efficiency of secondary production. Primary production is the result of subtracting row 3 from row 1 implying that whatever is not produced in primary will be produced in secondary production. Similarly, tertiary recovery is obtained from subtracting row 3 to row 2.

The standard deviation of the randomly generated parameters (rows 1-3) was set at 5% to avoid negative numbers for primary and tertiary recovery.

6.4.2 Results

The results are presented following the same structure as the independence case. Refer to **Table 26** and **Table 27** for the oil price, original oil in place, discount rate and yearly operating cost.

6.4.2.1Myopic and Life Cycle Optimization for Primary Recovery

Figures 36 and 37 show similar numbers than the independence case: the same project life, slightly larger NPV and recovery efficiency (see section 6.3.2.1 for plot interpretation).

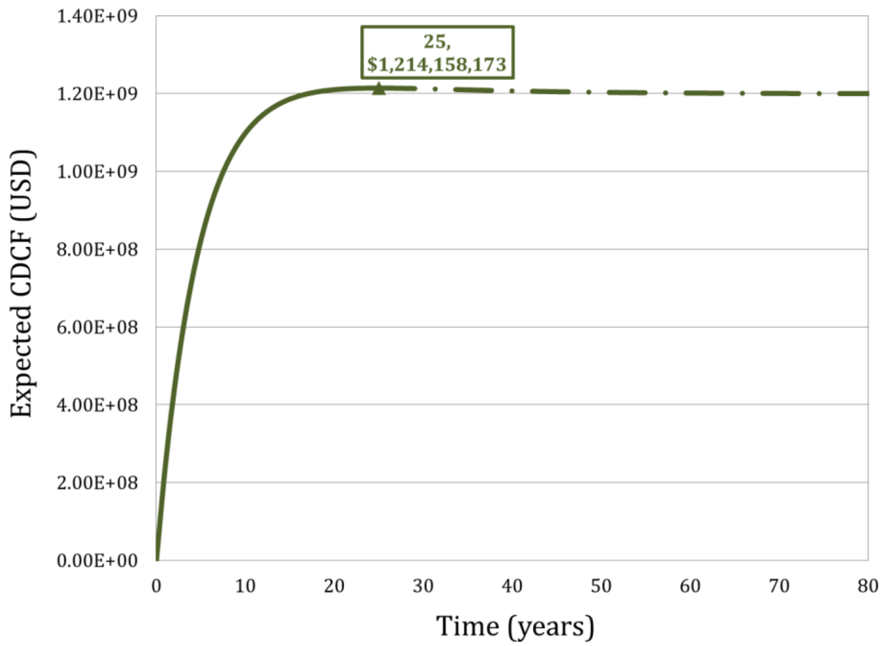

Figure 36: Expected cumulative discounted cash flow as a function of time (primary production)

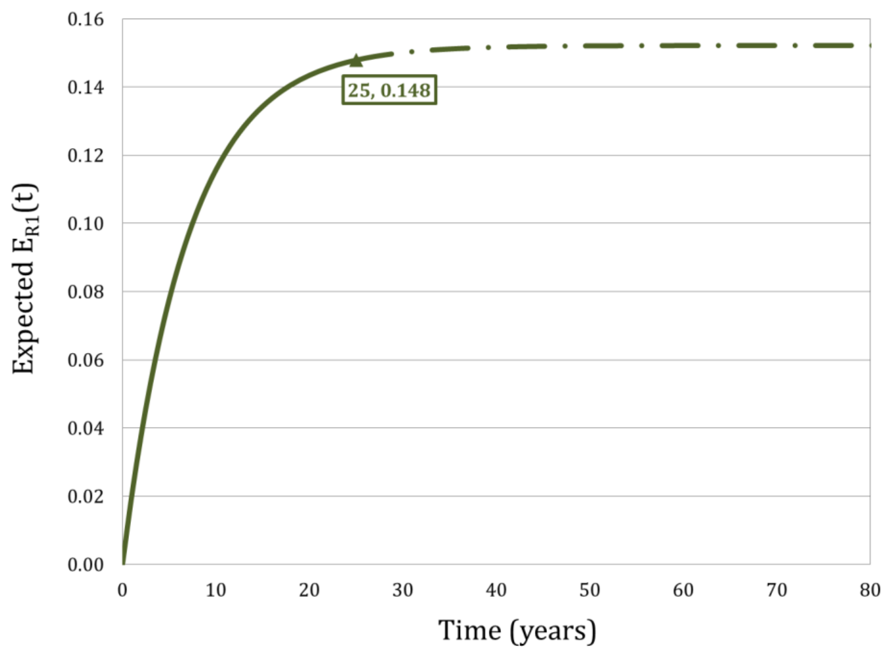

Figure 37: Expected recovery efficiency as a function of time (primary production)

6.4.2.2 Myopic and Life Cycle Optimization for Primary and Secondary Recovery

Table 32 summarizes the results for two phases, assuming dependency between primary and secondary production. **Figure 38** and **Figure 39** show the graphical representation of these numbers. There is no significant difference with the independent case: same project life, slightly larger NPV and recovery efficiency (refer to section 6.3.2.2 for plot interpretation).

Table 32: Myopic versus life cycle optimization for two phases

Expected values	Myopic Optimization	Life Cycle Optimization
NPV ($bn)	**$1.44**	**$1.74**
t_{Life} (years)	**49**	**32**
Cummulative Recovery Efficiency	**0.329**	**0.284**

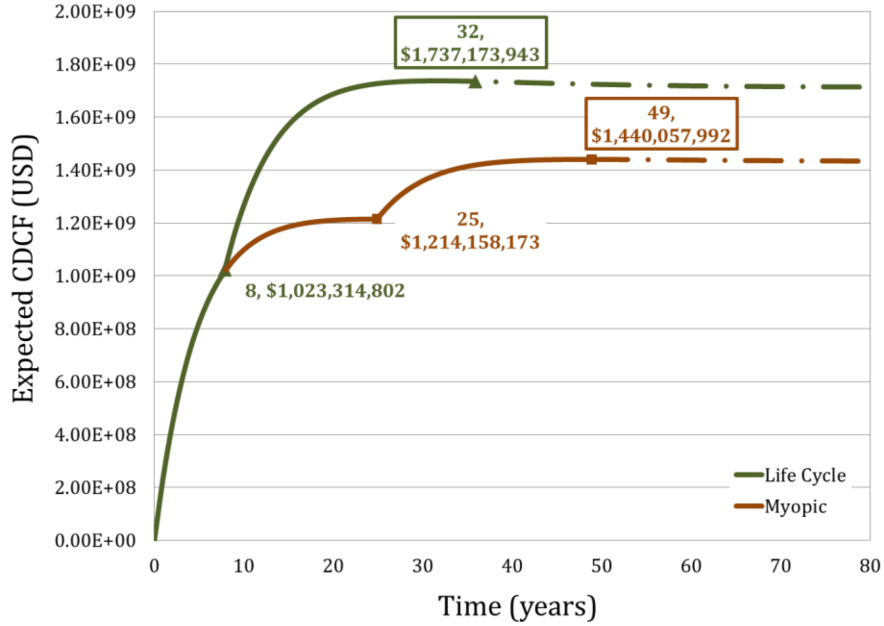

Figure 38: Expected cumulative discounted cash flow as a function of time (primary and secondary production)

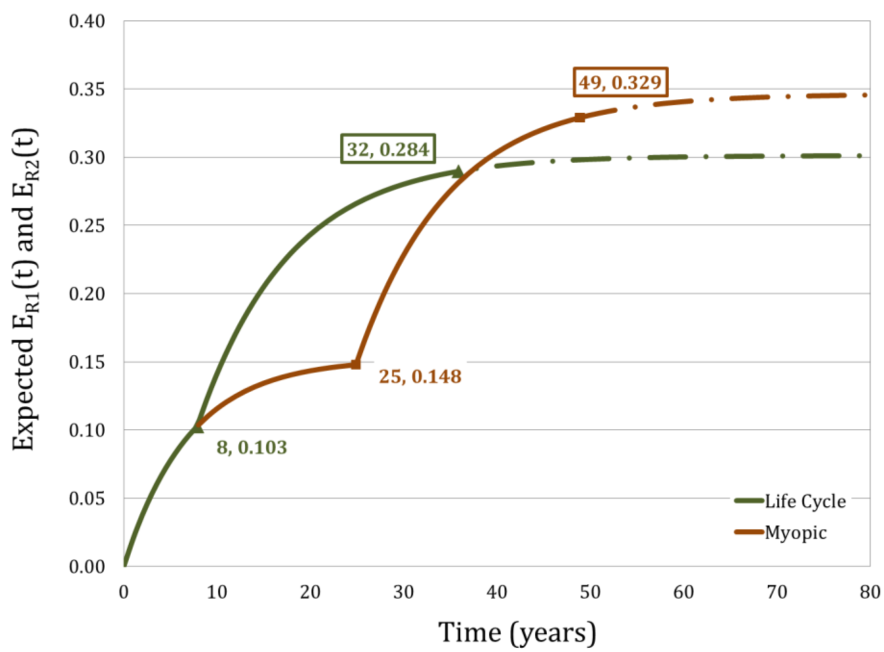

Figure 39: Expected recovery efficiency as a function of time (primary and secondary production)

6.4.2.3Myopic and Life Cycle Optimization for Primary, Secondary and Tertiary Recovery

Table 33 presents the results for three phases assuming dependency between the primary and secondary production and secondary and tertiary production. The graphical representation is in **Figure 40** and 41. We observe the same project life as the independent case, slightly larger NPV and recovery efficiency.

Table 33: Myopic versus life cycle optimization for three phases

Expected values	Myopic Optimization	Life Cycle Optimization
NPV ($bn)	$1.47	$1.89
t_{Life} (years)	63	34
Cummulative Recovery Efficiency	0.427	0.336

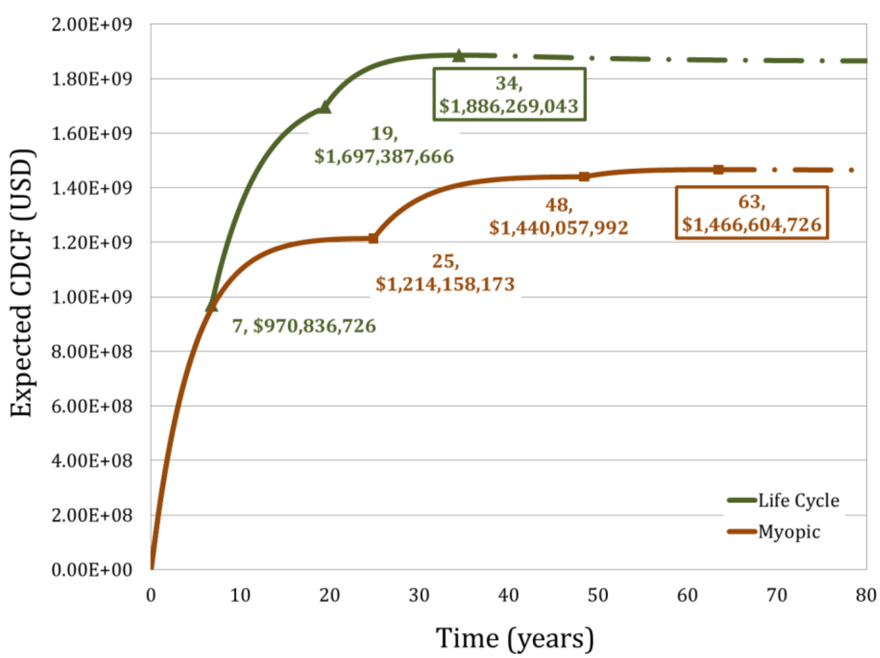

Figure 40: Expected cumulative discounted cash flow as a function of time (primary, secondary and tertiary production)

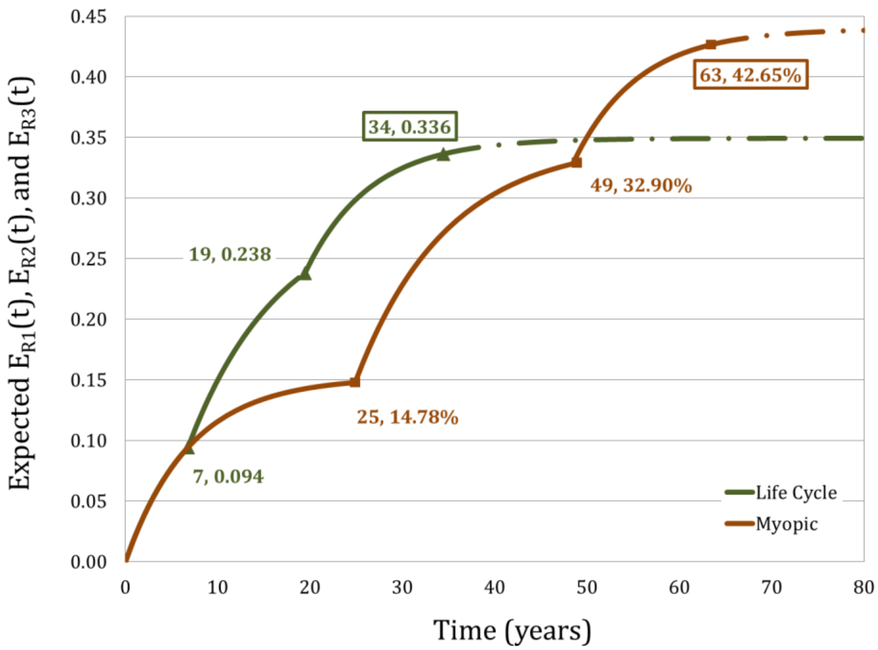

Figure 41: Expected recovery efficiency as a function of time (primary, secondary, and tertiary production)

6.4.3 NPV and Optimal Times Results

6.4.3.1 Comparing Myopic and Life Cycle Results

We see the same trends for the average NPV and t_{Life} obtain from the different iterations.

Figure 42 summarizes the results of the NPV and **Figure 43** compares the life of the project when one, two or three phases are available. As in previous sections the NPV is slightly larger assuming dependence.

When averaging the project life the values are also vaguely larger than the independent case.

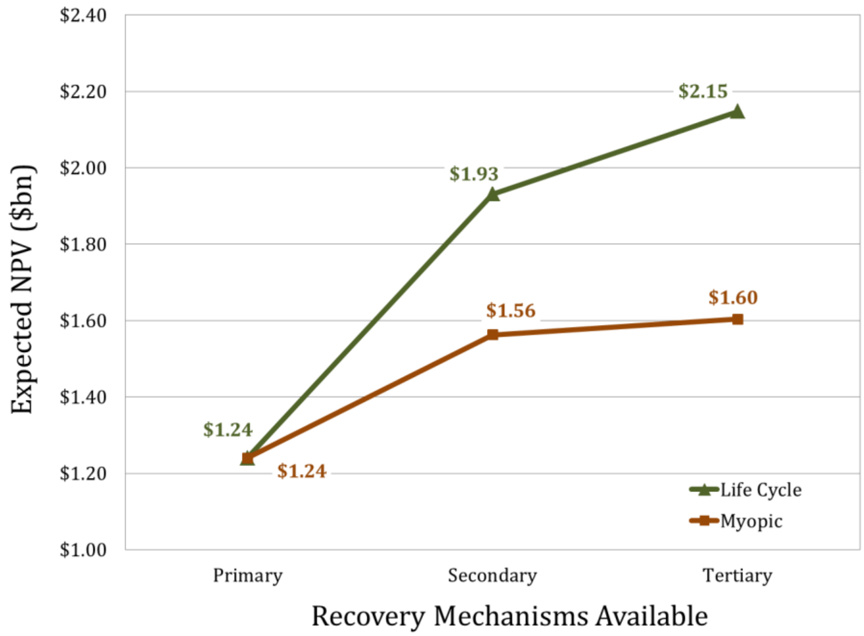

Figure 42: Expected NPV for life cycle and myopic optimization

Figure 44 illustrates the time assigned to each phase for the life cycle optimization. The general pattern is the same as the one observed for the independent case with small variations (refer to section 6.3.3.1 for the interpretation of the graph).

Figure 45 shows the average life of the project and the time allocated to each recovery phase for myopic optimization. We observe immaterial changes with respect to the independent case.

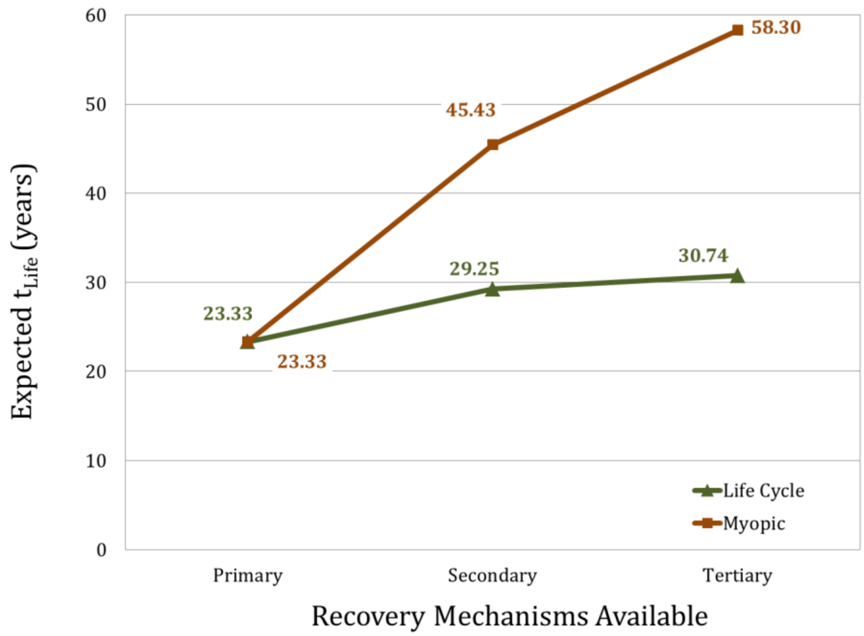

Figure 43: Expected life of the project for life cycle and myopic optimization

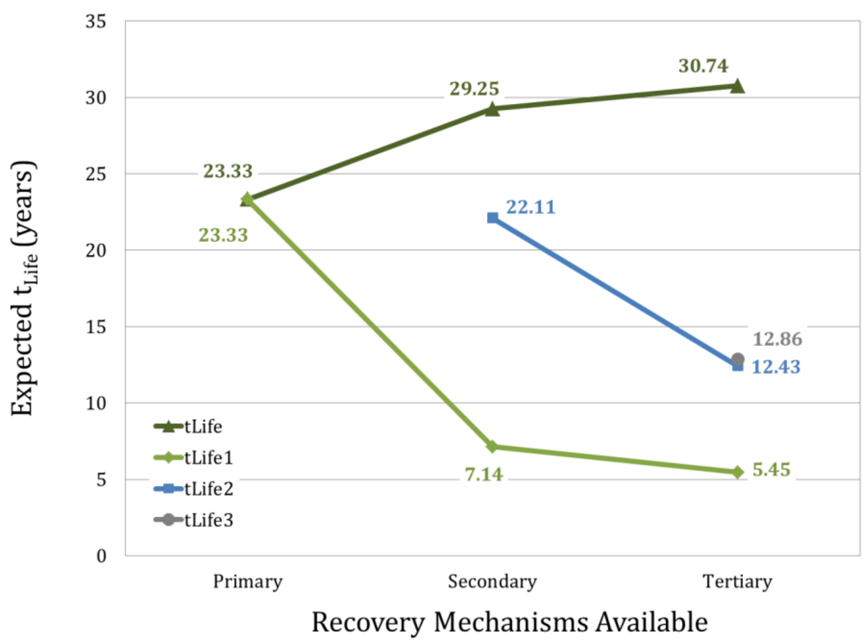

Figure 44: Expected life of the project and t_{Lifei} per recovery phase for life cycle optimization

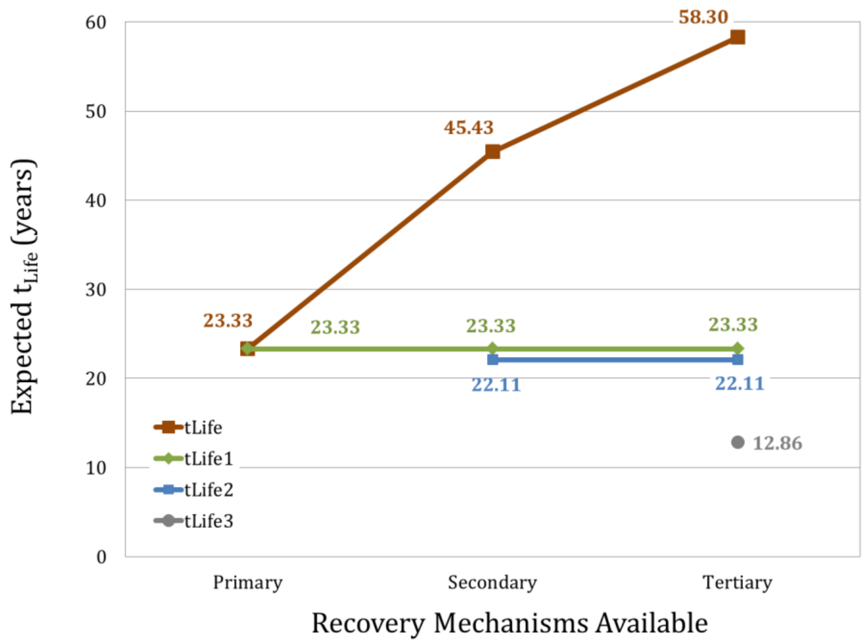

Figure 45: Expected life of the project and t_{Lifei} per recovery phase for myopic optimization

6.4.3.2 Analyzing Correlation Coefficients of Optimized Key Parameters

Table 34 presents the linear correlation coefficients between the uncertainties, the decision variables and the objective function. We assume dependence on any pair of variables with a correlation coefficient greater or equal to |0.4| (in bold).

We evaluate first the parameters that influence the NPV. Positive coefficients are related to theoretical ultimate recovery efficiency for primary production and in a lesser degree to tertiary production. Negative coefficients apply to any variables associated with secondary recovery. These results can be explained by the nature of the dependency generated between primary and secondary recovery. It is also logical to think that the NPV is larger when primary recovery is efficient as the operating costs associated with this recovery mechanism are smaller.

The dependencies observed for decision variables follow the same line of reasoning:

- The time devoted to primary recovery shows a strong dependency with the theoretical ultimate recovery efficiency for primary production. On the contrary, the more efficient secondary production is the less time we allocate to primary recovery.
- Similarly, the time assigned to secondary recovery depends largely on the theoretical ultimate recovery efficiency for secondary production. We observe negative correlation coefficients for primary and tertiary recovery. Again we have imposed this result when assuming dependency between recovery efficiencies.

Table 34: Linear correlation coefficients assuming dependence for life cycle optimization

	E_{R1}^{∞}	τ_1	ΔE_{R2}^{∞}	τ_2	ΔE_{R3}^{∞}	τ_3	t_{Life1} (years)	t_{Life2} (years)	t_{Life3} (years)	t_{Life} (years)	NPV_{LC} ($)
E_{R1}^{∞} (fraction)	1	0.02	**-0.69**	0.01	**0.42**	0.00	**0.82**	**-0.48**	0.38	0.26	**0.59**
τ_1 (years)		1	-0.05	-0.02	0.04	0.04	-0.04	-0.04	0.05	-0.02	-0.36
ΔE_{R2}^{∞} (fraction)			1	-0.03	**-0.65**	0.01	**-0.63**	**0.72**	**-0.58**	-0.08	-0.26
τ_2 (year)				1	0.01	-0.01	0.29	0.11	0.00	0.28	-0.37
ΔE_{R3}^{∞} (fraction)					1	-0.02	0.21	**-0.82**	**0.80**	-0.04	0.38
τ_3 (year)						1	0.08	0.21	0.36	**0.60**	-0.11
t_{Life1} (years)							1	-0.29	0.30	**0.50**	0.22
t_{Life2} (years)								1	**-0.65**	0.35	**-0.44**
t_{Life3} (years)									1	0.38	0.20
t_{Life} (years)										1	-0.19
NPV_{LC} ($)											1

Note: In bold are the correlation coefficients ≥|0.4|

- The time assigned to tertiary recovery is determined by the ultimate recovery efficiency of tertiary production.
- We expect longer project life when the time constant for production for tertiary recovery is large.

6.5 Conclusion

This chapter introduces uncertainty propagation through Monte Carlo simulation. Starting from the base case we adopted new ranges for ultimate recovery efficiency and the time constant for production and generated inputs randomly. Once the values were generated we performed a deterministic optimization myopically, by maximizing the net present value per recovery phase; and with a life cycle approach, or maximizing the net present value over the life of the project.

In contrast to traditional depletion strategies, where the optimization is done myopically the results of a life cycle approach show:

- The net present value is consistently larger when maximized over the life of the project. The percent improvement of the life cycle versus the myopic optimization averages 37%.
- Severe reduction of the project life when the NPV is maximized over the long-term. This reduction can halve the t_{Life} of the myopic solution.
- Total recovery efficiency decreases as a result of life cycle optimization. The difference fluctuates between 4 and 10%.
- The only differences detected between dependent and independent recovery efficiencies are attributed to the dependency itself. No material changes can be reported in terms of NPV, life of the project or recovery efficiency.
- Large NPV values are associated with relatively high recovery efficiencies in at least one of the production stages, preferably primary recovery.
- The time constant for production τ, has an extensive impact on the net present value and the optimal time assigned per recovery phase. Large recovery efficiencies combined with large time constants for production

result in small NPV values. Additionally, the time devoted to each recovery phase is inversely proportional to τ.

- The time assigned to the first recovery mechanism, whether primary, secondary or tertiary is decisive in maximizing total returns.

Chapter 7: Conclusions and Future Work

7.1 Review of the Research

Optimization of field operations is a subject that has been explored by many authors. Numerous approaches have been suggested to maximize profits by scheduling production over a limited period. These short to medium-term strategies have frequently focused on one recovery phase where the optimization does not consider the potential of future production phases. Industry practice has commonly associated long-term strategies with drilling programs, facilities and field development; however, there is little evidence for successful applications of optimum reservoir exploitation strategies over the life of the project.

The primary goal of this research was to provide a systematic optimization method to maximize the economic potential of the reservoir from discovery to field abandonment. Optimization requires defining an objective function and the corresponding decision variables. The objective function selected was the net present value and the decision variables were the time that should be assigned to each recovery phase to maximize the objective function or NPV. To optimize on the reservoir performance, the recovery efficiency history was introduced into the objective function assuming an exponential decline model.

Both numerical and analytical solutions were derived and used throughout the research allowing for greater flexibility in the analysis.

Once the optimization model was defined, we used a base case to test it against actual field data. Next, we performed a sensibility analysis to determine how the parameters influence the objective function and the decision variables.

Finally, the robustness of the model was evaluated using a stochastic approach.

7.2 CONCLUSIONS

We have proved the economic potential of a life cycle optimization for practical reservoir management through the base case and case study presented. Our research demonstrates the importance of an integrated strategy and the influence of the appropriate choice of decision variables (in our case time allocated to each production phase) over the net present value of the project.

The next two sections show the conclusions drawn from the study. The first section, general conclusions, includes a comparison of the main results obtained from myopic or short to medium-term strategies, and life cycle optimization. The second section, key parameters, emphasizes the impact of some of the variables in the model on the net present value and the time devoted to each recovery phase.

7.2.1 General Conclusions

In contrast to traditional depletion strategies, where the optimization is done in the short medium-term by maximizing the NPV at each recovery phase, the results of this life cycle optimization show:

- *Greater economic efficiency of the life cycle approach.* The net present value is consistently larger when maximized over the life of the project. These results are prevalent for both the deterministic and stochastic approach where more than 10,000 different trials were maximized. The percent improvement averages 37%.
- *Severe reduction of the project's life when the NPV is maximized over the long-term.* The life cycle optimization generates shorter project life than the myopic optimization for all the cases considered. This reduction can be up to half the value of the myopic solution.

- *Total recovery efficiency decreases as a result of life cycle optimization.* Long-term optimization invariably results in a reduction of the percentage of original oil in place recovered. The difference fluctuates between 4 and 10%. Recall that in the life cycle approach, production is economically viable beyond the optimal time assigned per recovery phase to maximize the net present value. Past that point, the net present value and the recovery efficiency become competing criteria. Additional production will increase recovery efficiency to the detriment of net present value and vice versa.

  A life cycle optimization provides the production schedule of a reservoir for a given cost structure. Although t_{Life} indicates the time in which a reservoir should be abandoned these results are inherently associated with the operating costs of the company operating a field. A different operator specialized in depleted fields might provide a more efficient cost structure that justifies further production while increasing the net present value. The fact that more oil is left behind with the life cycle approach could justify a larger value when selling the asset.

7.2.2 Key Parameters

From the analysis of the stochastic simulation the following conclusions can be drawn:

- Large NPV values are associated with relatively high recovery efficiencies in at least one of the production stages.
- The time constant for production τ has an extensive impact on the net present value and the optimal time assigned per recovery phase. Large recovery efficiencies combined with large time constants for production

result in small NPV values. Additionally, the time devoted to each recovery phase is inversely proportional to τ.

- The time assigned to the first recovery mechanism, whether primary, secondary or tertiary is decisive in maximizing returns.

7.2.3 Other Considerations

Optimization of the decision variables, t_{Life1}, t_{Life2} and t_{Life3}, is not affected by the following parameters:

- Introduction of new technologies, as long as adding a production phase is an option.
- Oil price, assuming the inflow offsets the outflow.
- Capital expenditure, if the capital costs associated with a new production phase are justified in terms of future income.

Although these parameters have a minor impact, if any, on the decision variables, they can cause large fluctuations in the net present value.

7.3 RECOMMENDATIONS FOR FUTURE WORK

The optimization model presented in this study uses well known concepts and is relatively simple to implement making this method both attractive and reliable.

Our goal was to develop a simplified model that can be readily used in the industry throughout the research. While some adjustments in the objective function might be required to fit a particular reservoir behavior or expenditure profile, this will come at the cost of increasing the complexity of the mathematical model. In particular, the objective function may be difficult to formulate analytically.

Before adding further complexity to the existing formulation, it is worth considering the balance between the details of reservoir behavior and economic representation. When solving decision problems of a broader nature, we recommend taking a global approach where the reservoir description can be simplified.

Future work should address the following issues:

- We have assumed that the incremental secondary recovery efficiency is independent of the switching time. We recommend deriving a function that relates secondary recovery to switching time.
- Develop a simpler analytical solution for three phases. Along this line it would be advisable to find broad principles that capture rules of thumb or key factors affecting the decision variables.
- Introduce optimization using dynamic programming. Although dynamic programming would require additional computational effort, it would allow the decision in one step to influence subsequent time steps. Second, dynamic programming would facilitate the introduction of uncertainty to key parameters such as operating costs and original oil in place.
- Extend the existing model from an exponential to a hyperbolic decline.
- Customize operating costs and capital expenditure according to a given corporation cost structure forecasts. Some industry experts suggest that operating costs decrease within a recovery phase as they become more efficient in their operations. Although these variable costs can easily be reflected in the numerical approach the analytical solution might be difficult to derive.

APPENDIX A: CASE STUDY

This appendix contains a case study from a field in Seminole, Texas. Waterflooding began in 1969 and CO_2 injection started in 1986. Operations continue until 2009. No data is available for primary recovery.

A Field in West Texas

A.1 Data

Table 35 contains the reservoir parameters obtained from the field data.

Table 35: Case study reservoir parameters

Parameters	Values
E_{R2}^{∞} (fraction)	0.105
τ_2 (years)	5
ΔE_{R3}^{∞} (fraction)	0.096
τ_3 (years)	8.3
N (bbls)	2,800,000,000
t_{Life2} (years)	17
t_{Life3} (years)	23

Table 36 shows the yearly operating cost per recovery phase (refer to section 2.5.3.1 on how to estimate operating cost per year). We assume $55 per barrel and a discount rate in real terms of 7%.

We use the analytical approach for the myopic and life cycle optimization and the numerical solution for plotting the results of the myopic, case study and life cycle optimization.

Table 36: Yearly operating cost per recovery phase

Recovery Phase$_i$	Opex ($/bbl)	$\$_{Opexi}$
Secondary Pattern	7	$68,429,958
Tertiary	12	$117,308,499

A.2 Comparing Myopic, Case Study and Life Cycle Optimization

A.2.1 Net Present Value

Table 37 summarizes the results for the three cases. We observe that the case study approximates a myopic optimization. The results are in accordance with what we have obtained in the deterministic and stochastic approach. The life cycle optimization leads to a larger NPV and a shorter life of the project. The recovery efficiency is reduced when maximizing over the life of the project.

Table 37: Myopic, case study and life cycle results

	Myopic Optimization	Base Case Optimization	Life Cycle Optimization
NPV ($bn)	**$12.97**	**$13.30**	**$14.34**
t_{Life} (years)	**42**	**40**	**32**
Cummulative Recovery Efficiency	**0.192**	**0.191**	**0.177**

Figure 46 shows the cumulative discounted cash flow as a function of time. The life cycle optimization results in an 8% improvement over the case study and 11% over the myopic optimization.

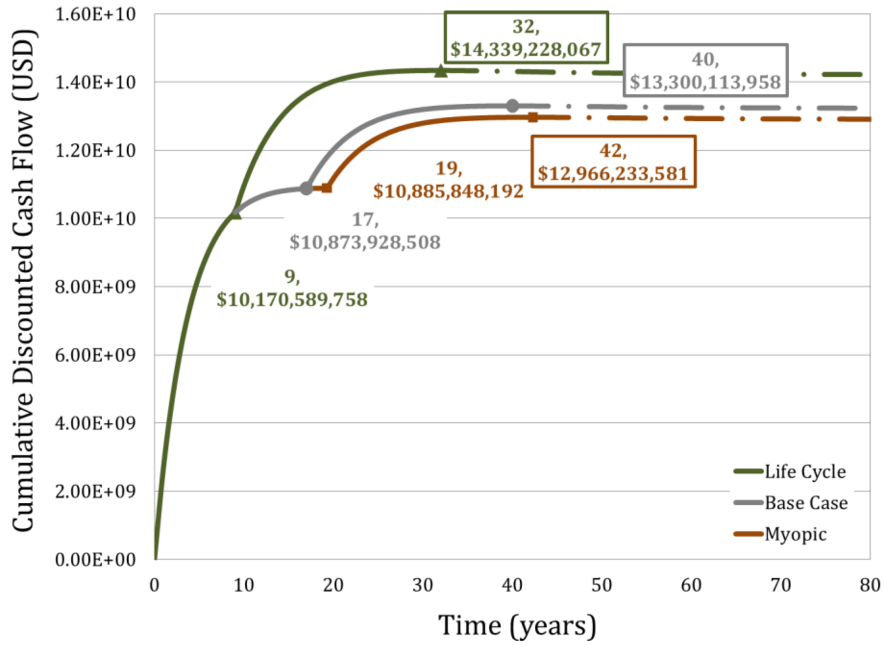

Figure 46: Cumulative discounted cash flow as a function of time

A.2.1 Recovery Efficiency

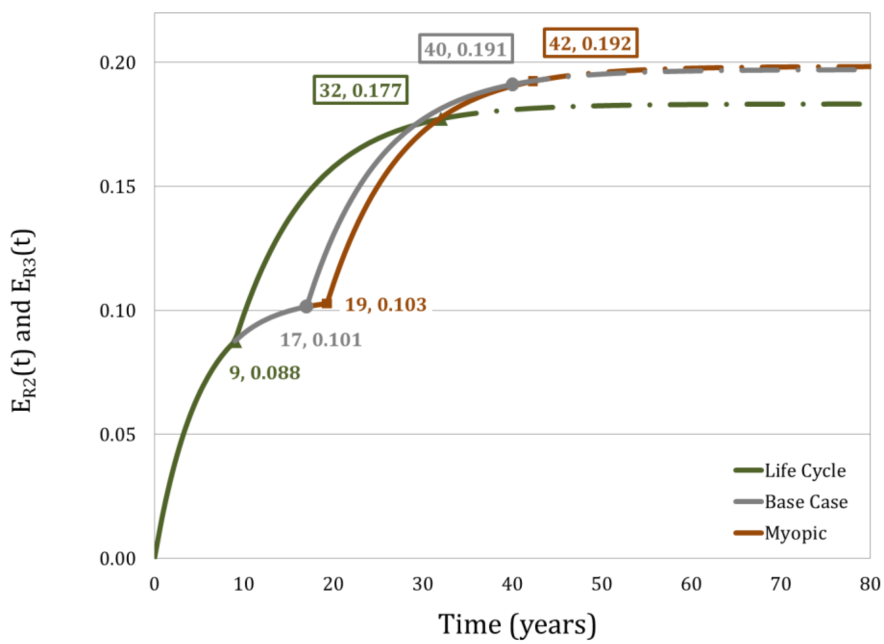

Figure 47: Recovery efficiency as a function of time

Figure 47 shows the recovery efficiency as a function of time. The reduction in the percentage of oil recovered from the life cycle solution is the result of moving to tertiary before the economic limit is reached. Myopic, case study and life cycle solution lead to the same incremental recovery efficiency for CO_2 injection (8.96%).

A.2.2 Life of the Project

Figure 48 illustrates the time assigned to secondary and tertiary recovery. We observe that the operator kept production for 8 additional years compare to the life cycle approach. Those 8 years only provided an extra 1.39% of the final recovery efficiency and resulted in a $1.04 bn reduction in the NPV of the project.

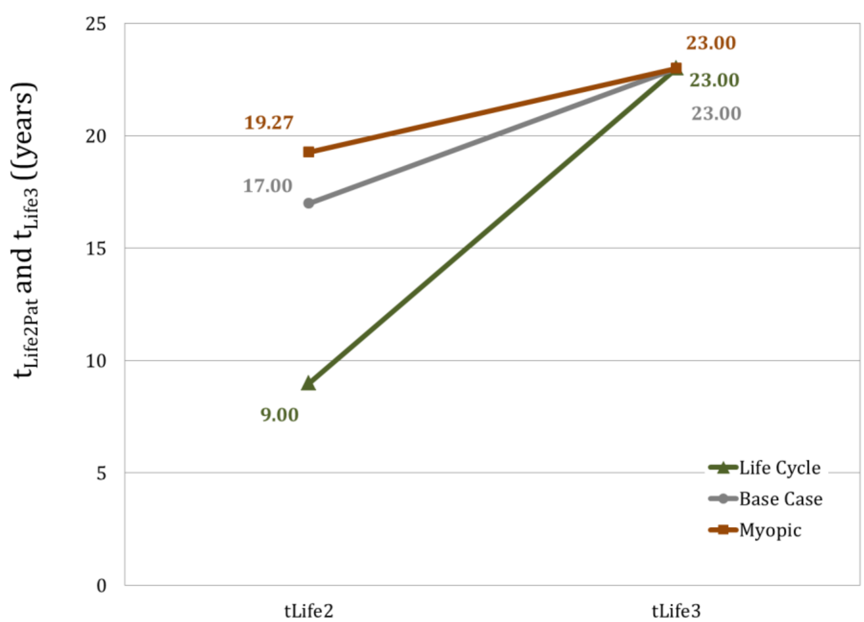

Figure 48: t_{Life} per recovery phase

A.3 DISCUSSION

This case study proves the benefits of the life cycle optimization for an onshore field in Seminole, Texas. We maximize the NPV by assigning the optimal time that should be allocated to secondary recovery. The results show that production continued 8 additional years compare to the life cycle solution leading to a $1.04 bn reduction of the NPV.

References

Andonyadis, P., "Decision Support for Enhanced Oil Recovery Projects," MS Thesis, the University of Texas at Austin, August 2010.

Aronofsky, J. S., "Optimization Methods in Oil and Gas Development," SPE paper 12295 available from SPE, Richardson, Texas, 1983.

Barua, J., Prescott, T., and Haldorsen, H.H., "Financial and Technical Decision Making for Surfactant Flooding," SPE paper 15074, presented at the 56th Annual California Regional Meeting of the SPE, Oakland, California April 1986.

Bickel, J. E., and Smith, J. E., "Optimal Sequential Exploration: A Binary Learning Model," Decision Analysis, 2 (1): 16-32, March 2006.

Bickel, J. E., Smith, J. E., and Meyer, J. L., "Modeling Dependence Among Geologic Risks in Sequential Exploration Decisions," SPE paper 102369, presented at the SPE Annual Technical Conference and Exhibition, San Antonio, Texas, September 2006.

Bickel, J. E., Lake, L. W., and Lehman J., "Discretization, Simulation and Swanson's Mean," under review at SPE Economics & Management, 2010.

Brokmeyer R. J., Borling, D. C., and Pierson, W. T., "Lost Soldier Tensleep CO_2 Tertiary Project, Performance Case History; Bairoil, Wyoming," presented at the Permian Basin Oil and Gas Recovery Conference, Midland, Texas, March 1996.

Carroll III, J. A., and Horne, R.N., "Multivariate Optimization of Production Systems," SPE paper 22847, presented at the SPE Annual Technical Conference and Exhibition, Dallas, Texas, October 1991.

Howard, R. A., "Decision Analysis, Section 1: Possibilities and Probabilities," 2006.

Lake, L. W., and Walsh, M. P., "A Generalized Approach to Primary Hydrocarbon Recovery," Elsevier, Amsterdam, The Netherlands, 2003.

Lang, Z. X., and Horne, R. N., "Optimum Production Scheduling Using Reservoir Simulators: A Comparison of Linear Programming and Dynamic Programming Techniques," SPE paper 12159, presented at the 56th Annual Technical Conference and Exhibition, San Francisco, California, October 1983.

Lawal, A. A., “Applications of Sensitivity Analysis in Petroleum Engineering,” Master’s Thesis, The University of Texas at Austin, May 2007.

Nystad, A. N., “Petroleum Reservoir Management: A Reservoir Economic Approach,” SPE paper 13833 available from SPE, Richardson, Texas, 1984.

Saputelli, L., Garibald, L., Smyth, J., Ungredda, A., Rodriguez, J., and Cullick, A.S., “How Integrated Field Studies Help Asset Teams Make Optimal Field Development Decisions,” SPE paper 110250, presented at the SPE Western Regional and Pacific Section AAPG Joint Meeting, Bakersfield, California, March 31-April 2 2008.

Saputelli, L., Mijares, G., Rodriguez, J.A., Sankaran, S., Giribaldi, L., and Revana, K.,“Managing Risk and Uncertainty in the Visualization of Production Scenarios,” SPE paper 107562, presented at the 2007 Hydrocarbon Economics and Evaluation Symposium, Dallas, Texas, April 2007.

Sarma, P. and Chen, W.H., “Applications of Optimal Control Theory for Efficient Production Optimization of Realistic Reservoirs,” International Petroleum Technology Conference paper 12480 presented at the IPTC, Kuala Lumpur, December 2008.

Sarma, P., Chen, W.H., and Durlofsky, L.J., “Production Optimization With Adjoint Models Under Nonlinear Control-State Path Inequality Constraints,” SPE paper 99959, presented at the 2006 SPE Intelligent Energy Conference and Exhibition, Amsterdam, The Netherlands, April 2006.

Sarma, P., Durlofsky, L.J., and Aziz, K., “Efficient Closed-loop Production Optimization Under Uncertainty,” SPE paper 94241, presented at the SPE Europec/EAGE Conference, Madrid, Spain, June 2005.

See, B. A., and Horne, R. N, “Optimal Reservoir Production Scheduling by Using Reservoir Simulation,” SPE paper 11133, presented at the SPE Annual Technical Conference and Exhibition, New Orleans, Louisiana, September 1982.

Simmons, J. F., and Falls A. H., “The Use of Dimensionless Scaling Methods To Predict Field-Scale Tertiary EOR Project Performance,” presented at the SPE Annual Technical Conference Exhibition, Dallas, Texas, October 2005.

Singh, V. P., Jain, S. K., and Tyagi, A., "Risk and Reliability Analysis: A Handbook for Civil and Environmental Engineers," ASCE Press/America, Reston, Virginia, 2007.

Vasantharajan, S., Al-Hussainy, R., and Heinemann, R.F., "Applying Optimization Technology in Reservoir Management," SPE paper 87836, 82-88, May 2006.

Wences, A., and Tomas, M., "Risk Analysis in the Determination of the Best Exploration Strategy Using a Calibrated Reservoir-Simulation Model," SPE paper 94804, presented at the SPE Latin American and Caribbean Petroleum Engineering Conference, Rio de Janeiro, Brazil, June 2005.

Xu, L., "Application of Real Options to the Valuation and Decision Making in the Petroleum E&P Industry," Ph.D. Dissertation, The University of Texas at Austin, 2010.